Electrical Charge and Field

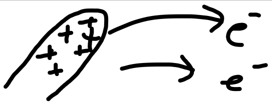
The physical property of matter that causes it to experience as a force when placed in an electromagnetic field is called charge.

- field – effect.
- electro magnetic field – Electric + magnetic effect.
  where electric and magnetic boths effects are considered.

- charge ⟶ Scalar quantity. ⟶ $q = i.t.$
- charge Unit – 'C' ⟶ $i = \frac{dq}{dt} \Rightarrow q = it \rightarrow A.S.$

Types of charges.

Positive charge. Negative charge.




An object. Attends positively charged by losing electrons, while other can obtain negative charge by gaining electrons.

Laws of charges.

1. Opposite charges attract.
2. Similar charges repel.

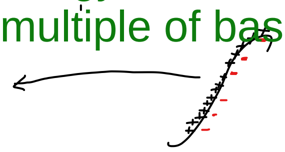
-Attraction or repulsion is a vector.

-Study of force field and potential when at rest is called 'electrostatics'.

-To know the amount of force we use 'Coulombs law'.

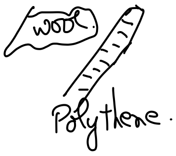
Properties of charges.

1. Charges always reset on the surface of the charged object.
2. Charges are always added.
3. Charges can never be created, not destroyed - "Conservation of energy".
4. Charge on a body can be expressed as an. Integral multiple of basic unit of charge. $\boxed{q = \pm ne}$



Q A polythene piece rubbed with wool is found to have a -ve charge of 3×10^{-7} C. (1) Find the number of electron transfered from which to which? (2) Is there a transfer of mass from wool to polythene?

Sol



(1) $q = -3\times10^{-7}$ C.

$e = -1.6\times10^{-19}$ C.

→ No of electrons transfered from wool to Polythene

$q = ne. \qquad n = \frac{q}{e}$

$n = \frac{-3\times10^{-7}}{-1.6\times10^{-19}} = 1.875\times10^{12}$ Ans

(2) Yes, there is.

$m_e = 9\times10^{-31}$ Kg. $\qquad m = 1.875\times10^{12}\times 9\times10^{-31}$.

$m = 16.875\times10^{-19}$ Kg. Ans

Q A Cu slab of mass 2g contains 2×10^{22} atoms. The charge on the nucleus of each atom is 29e. What fraction of the electrons must be removed from the sphere to give it a charge of $+2\mu C$?

Sol Total charge of electrons $= 29e = 29\times(\text{No of electron}) = 29\times(\text{No of Atoms})$
$= 29\times2\times10^{22}$.

$q = ne. \quad n = \frac{q}{e} = \frac{2\times10^{-6}}{1.6\times10^{-19}} = 1.25\times10^{+13}$ electrons must be removed.

Fraction of electrons removed $= \frac{1.25\times10^{+13}}{\text{Total electron}} = \frac{1.25\times10^{+13}}{29\times2\times10^{22}} = 0.02155\times10^{-9}$

$= 2.155\times10^{-11}$. Ratio

Ans

Q If a body gives out 10^9 electrons every second, how much time is required to get a total charge 1 C from it.

Sol In 1 s — 10^9 electrons are given out.

∴ charged → $10^9\times1.6\times10^{-19}$ C. $= 1.6\times10^{-10}$ C. in 1 s.

1.6×10^{-10} C charge given in 1 S

— 1C charge given in $\frac{1}{1.6\times10^{-10}}$ s $= \frac{10^{10}}{1.6} = 198.18$ years.

Q How much positive and negative charge is there in a Cup of Water?

Sol We assume a cup to have 250g.

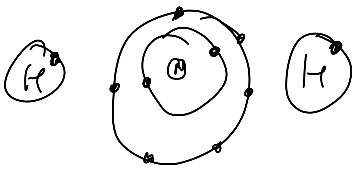
Water → Molecular mass is 18g.

No of molecules present in 18g of H_2O = Avogadro's No = 6.02×10^{23} particles.

$\therefore$ —— 1 g of H_2O —— $= \dfrac{6.02 \times 10^{23}}{18}$

—— 250g H_2O —— $= \dfrac{6.02 \times 10^{23}}{18} \times 250 = 8.337 \times 10^{24}$.

In one molecule of $H_2O \rightarrow 2 + 8 = 10$ electrons.



Total No of electrons $= 10 \times 8.337 \times 10^{24}$

$= 8.337 \times 10^{25}$

$\therefore$ +ve and −ve charges are same. (NUTRAL)

Total charge $= 8.337 \times 10^{25} \times 1.6 \times 10^{-19}$.

$= 1.33 \times 10^{7}$ C.

Q A sphere of lead of mass 10g has a net charge -2.5×10^{-9} C.

① Find the no of excess electrons on the sphere.

② How many excess electrons are per lead atom?

Atomic Number of Pb is 82 and Atomic mass is 207g/mole.

Sol $e = 1.6 \times 10^{-19}$ C.

① Net charge $= -2.5 \times 10^{-9}$ C.

$q = ne$.

$n = \dfrac{q}{e} = \dfrac{-2.5 \times 10^{-9}}{1.6 \times 10^{-19}} = 1.5 \times 10^{10}$ electrons.

Ans

② 10g of Pb will have $= \dfrac{10}{207} \times 6.02 \times 10^{23}$ atoms.

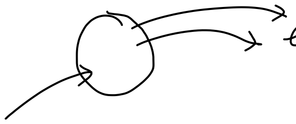
$= .291 \times 10^{23} = 2.91 \times 10^{22}$ atoms.

No of excess electrons per atom $= \dfrac{1.5 \times 10^{10}}{2.91 \times 10^{22}} = 5.36 \times 10^{-13}$ electrons.

Ans

Q A metal sphere has a charge of $-6\mu C$. When 5×10^{12} electrons are removed from the sphere, what would be the net charge on it.

Sol



electrons removed $= 5\times10^{12}$. which means it gain +ve charge.

$q = -6\mu C$.

$q = ne$.

$q = 5\times10^{12}\times1.6\times10^{-19}$.

$= 8\times10^{-7} = 0.8\times10^{-6}\,C$.

$q = +0.8\mu C$.

Net charge $= -6\mu + 0.8\mu = -5.2\mu C$.

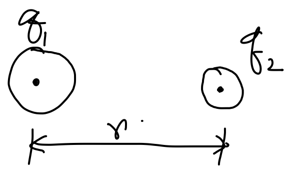
Ans

Coulomb's Law

→ why?? To know what is the amount of attraction and repulsion between two or more point charges.

Charge law – Similar charge → repels.

Opposite charge → Attraction → Force.



$F \propto \frac{q_1 q_2}{r^2}$

$F = K\frac{q_1 q_2}{r^2}$

$$\boxed{F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}}$$

"Force of attraction or repulsion between two stationary point charges is directly proportional to the product of charge and inversely proportional to the square of the distance between them".

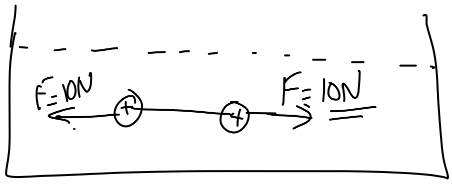
$K = \frac{1}{4\pi\varepsilon} = 9\times10^{9}\,Nm^2/C^2$.

$K = \frac{Fr^2}{q_1 q_2} \rightarrow Nm^2/C^2$.

ε = permittivity.

Unit – C^2/Nm^2

$$\boxed{\varepsilon = \varepsilon_0\varepsilon_r}$$



ε_0 = Absolute permittivity: $\varepsilon_0 = 8.85\times10^{-12}\,C^2/Nm^2$

ε_r = Relative permittivity

$\varepsilon_r = 1 \longrightarrow$ Air/Vacuum.

Q What is the force between two small charged sphere having charge of $2 \times 10^{-7} C$ and $3 \times 10^{-7} C$ placed 30 cm apart in air?

Sol

$$F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2} \Rightarrow 9 \times 10^9 \times \frac{2\times10^{-7} \times 3\times10^{-7}}{(30\times10^{-2})^2}$$

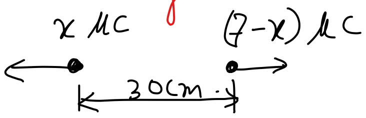
$$= \frac{\cancel{9}^{3} \times 2 \times \cancel{3}}{\cancel{30}_{10} \times \cancel{30}_{10}} \times \frac{10^{9-7-7}}{10^{-4}}$$

$$= 6 \times 10^{-2} \times 10^{-5+4} = 6 \times 10^{-3} N.$$

"2009"

Q The Sum of Two point charges is $7\mu C$. They repel each other with $F = 1N$ when kept 30cm apart in free space. Calculate the Value of each charge.

Sol



$F = 1N$.

$$\boxed{1\mu = 1\times10^{-6}}$$

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$$

$$F = \cancel{9}\times10^9 \times \frac{x(7-x)(10^{-6})^2}{\cancel{900}_{100} \times 10^{-4}} = 1.$$

$$10^{-12} \times 10^9 \times 10^{+2}\, x(7-x) = 1$$

$$7x - x^2 = \frac{1}{10^{11} \times 10^{-12}}$$

$$x^2 - 7x + 10 = 0$$

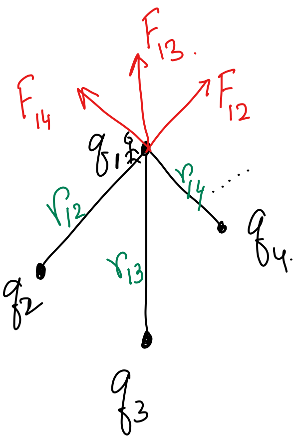
$$x^2 - 5x - 2x + 10 = 0$$

$$x(x-5) - 2(x-5) = 0$$

$$(x-2)(x-5) = 0$$

$$x = 2C \qquad x = 5C.$$

Q Forces between multiple charges. — Superposition.



Superposition.

$$F_{Total} = F_{12} + F_{13} + F_{14} + \cdots\cdots F_{1n}$$

$$= \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r_{12}^2} + \frac{1}{4\pi\varepsilon}\frac{q_1 q_3}{r_{13}^2} + \cdots\cdots \frac{1}{4\pi\varepsilon}\frac{q_1 q_n}{r_{1n}^2}$$

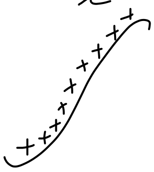
$$F_{Total} = \frac{q_1}{4\pi\varepsilon}\left[\frac{q_2}{r_{12}^2} + \frac{q_3}{r_{13}^2} + \cdots\cdots \frac{q_n}{r_{1n}^2}\right]$$

$$\boxed{F_{1i} = \frac{q_1}{4\pi\varepsilon}\sum_{i=2}^{n}\left(\frac{q_i}{r_{1i}^2}\right)}$$

Charge distribution.

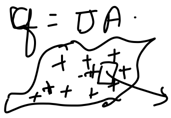
Line /Linear charge density.	Surface charge density.	Volume charge density.

$\lambda = \frac{q}{l}$ c/m.



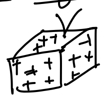
$q = \lambda l$.
$dq = \lambda dl$.
$q = \int \lambda dl$.

$\sigma = \frac{q}{A}$ c/m².
$q = \sigma A$.



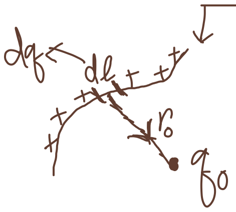
$dq = \sigma dA$
$q = \int \sigma dA$

$\rho = \frac{q}{\,}$.



$q = \rho V$
$dq = \rho dV$.
$q = \int \rho dV$

$$\boxed{\vec{F} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}\,\hat{a}_r}$$

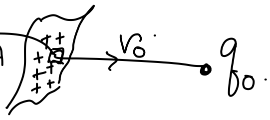


$\vec{dl} \rightarrow$ Vector

$$\vec{F} = \frac{1}{4\pi\varepsilon}\frac{q_0 \int \lambda dl}{r_0^2}\,\hat{a}_{r_0}$$
$$= \frac{q_0 \lambda}{4\pi\varepsilon}\int \frac{dl}{r_0^2}\,\hat{a}_{r_0}$$

$\vec{ds} \rightarrow$ Vector

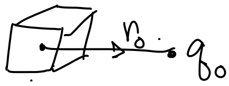
$q = \int \sigma dA$



$$\vec{F} = \frac{1}{4\pi\varepsilon}\frac{q_0 q}{r_0^2}\,\hat{a}_{r_0}$$
$$= \frac{q_0}{4\pi\varepsilon}\int \frac{\sigma dA}{r_0^2}$$
$$\vec{F} = \frac{q_0 \sigma}{4\pi\varepsilon}\int \frac{dA}{r_0^2}\,\hat{a}_{r_0}$$

$\rightarrow$ (Concept of Thumb only for Surface)

$dV \rightarrow$ Scalar.



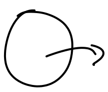
$$\vec{F} = \frac{1}{4\pi\varepsilon}\frac{q_0 q}{r_0^2}\,\hat{a}_{r_0}$$
$$\vec{F} = \frac{q_0}{4\pi\varepsilon}\int \frac{\rho dV}{r_0^2}\,\hat{a}_{r_0}$$
$$\vec{F} = \frac{q_0 \rho}{4\pi\varepsilon}\int \frac{dV}{r_0^2}\,\hat{a}_{r_0}$$

Q What charge would be required to electrify a sphere of radius 25cm so as to get a surface charge density of $\frac{3}{\pi}$ c/m²?

Sol $R = 25$ cm.

$\sigma = \frac{q}{A} \Rightarrow q = \sigma . A$.

$$= \frac{3}{\cancel{\pi}} \times 4\cancel{\pi} r^2 = 12(25\times10^{-2})^2 = \frac{\cancel{12}^{3}\times \cancel{25}\times \cancel{25}}{\cancel{100}_4\times\cancel{100}_4} = \frac{3}{4}\ c.$$

 $A = 4\pi r^2$

Q The radius of gold nucleus ($Z = 79$) is about 7×10^{-15} m. Assume +ve charge is distributed uniformly throughout the nuclear Volume, Find the Volume charge density.

Z – Atomic Number.

Sol

$q = Ze = 79 \times 1.6 \times 10^{-19}$

$R = 7 \times 10^{-15}$ m.

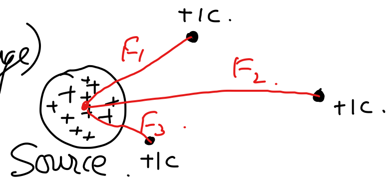
$$\rho = \frac{q}{V} = \frac{79 \times 1.6 \times 10^{-19}}{\frac{4}{3}\pi R^3}$$

$$= \frac{3 \times 79 \times 1.6 \times 10^{-19}}{4\pi (7 \times 10^{-15})^3} = 8.8 \times 10^{24}\ C/m^3$$

Electric field - (E)

effect → A type of force.

Unit positive charge (Test charge) $= +1C$.



$F_3 > F_1 > F_2$.

The force that a unit positive charge would experience if placed at that point.

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_0}{r^2}$$

$$E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon}\frac{q_1}{r^2}$$

$q_0 = 1\,C \rightarrow$ Test charge.

Electric field is force per unit charge.

$E = F/q$ Unit – N/C.

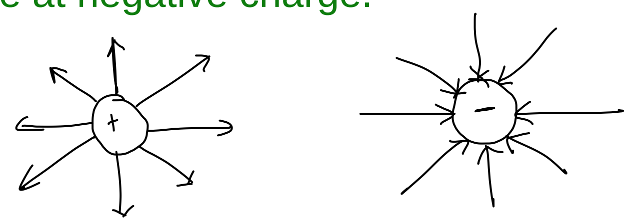
$$\boxed{E = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}}$$

$$\boxed{\begin{aligned} E &= \lim_{q_0 \to 0} \frac{F}{q_0} \\ &= \lim_{q_0 \to 0} \frac{1}{q_0}\left(\frac{1}{4\pi\varepsilon}\frac{q_0 q}{r^2}\right) \end{aligned}}$$

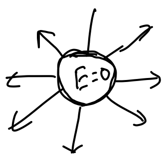
To get a visual effect, we draw imaginary curves called 'electric field lines'. But the field is real.

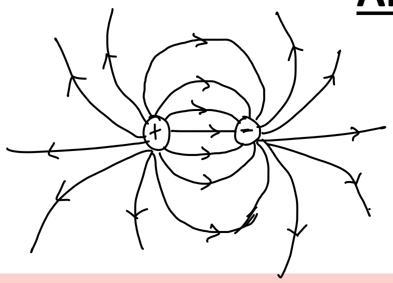
Rules

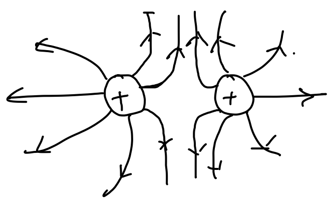
1. Lines will always come out from positive charge and will terminate at negative charge.



2. They are continuous.
3. No lines can cross each other.
4. The closeness of line tells the strength of electric field.
5. No lines are present inside the conductor.







Q A Conducting sphere of radius 10cm has an Unknown charge. If the electric field 20cm from the center of the sphere is 1.5×10^3 N/C and points radially inwards, then what is the net charge on the sphere?

Sol

$E = 1.5\times10^3$ N/C

| 20cm |

$$E = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}$$

$$q = 4\pi\varepsilon \times E \times r^2$$

$$= \frac{1}{9\times10^9}\times1.5\times10^3\times(20\times10^{-2})^2$$

$$= 6.67\times10^{9}\,C$$

Q The electrostatic force of repulsion between two +vely charged Ions carrying equal charge is 3.7×10^{-9} N when they are separated by a distance of 5°A. How many electrons are missing from each ion?

Sol

$F = 3.7\times10^{-9}$ N

$r = 5°A = 5\times10^{-10}$ m

$q_1 = +q$

$q_2 = +q$

$$F = \left(\frac{1}{4\pi\varepsilon}\right)\frac{q_1q_2}{r^2}$$

$$3.7\times10^{-9} = 9\times10^9\times\frac{q^2}{(5\times10^{-10})^2}$$

$$q^2 = \frac{3.7\times10^{-9}\times(5\times10^{-10})^2}{9\times10^9}$$

$$= \frac{3.7\times25}{9}\times10^{-9-20-9}$$

$$q^2 = 10.27\times10^{-38}$$

$$q = 3.2058\times10^{-19}\,C$$

(2) $q = ne$

$$n = \frac{q}{e} = \frac{3.2058\times10^{-19}}{1.6\times10^{-19}}$$

$$n = \cancel{\frac{3.2058}{1.6}} = 2$$

Q A ball A of 8g carry a +ve charge of 5×10^{-8}C. What must be the nature and magnitude of charge that should be given to a second ball B fixed 5cm below the former ball so that the ball is stationary??

Sol

A, F, $q_1 = 5\times10^{-8}$C, mg, 5cm, B

$F = mg \longrightarrow$ For ball A to stationary and B should have +ve charge (Nature)

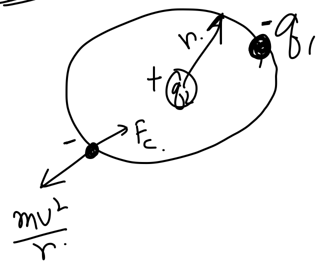
$$\frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2} = mg$$

$$q_2 = \frac{mgr^2\cdot4\pi\varepsilon}{q_1} = 8\times10^{-3}\times9.8\times(5\times10^{-2})^2 / 5\times10^{-8}\times9\times10^9 = 4.36\times10^{-7}C$$

+ve

Q A particle of mass m and charge $-q_1$ is moving around a charge q_2 in a circular path of Radius 'r'. Prove the period of Revolution $T = \sqrt{\frac{16\pi^3 \varepsilon_0 m r^3}{q_1 q_2}}$

Sol



$$\frac{mv^2}{\cancel{r}} = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^{\cancel{2}}}$$

$$v^2 = \frac{q_1 q_2}{4\pi\varepsilon r m}$$

$$V = \frac{D}{t} = \frac{2\pi r}{T}$$

$$v^2 = \frac{4\pi^2 r^2}{T^2}$$

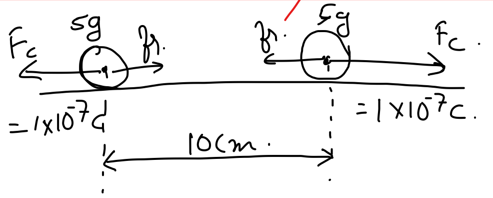
$$\frac{4\pi^2 r^2}{T^2} = \frac{q_1 q_2}{4\pi\varepsilon r m}$$

$$\therefore T^2 = \frac{16\pi^2 \varepsilon r^3 m}{q_1 q_2}$$

$$T = \sqrt{\frac{16\pi^3 r^3 m \varepsilon}{q_1 q_2}}$$

Q Two particle, each having a mass of 5g and charge 1×10^{-7}C, stay in limiting equilibrium on a horizontal table with a separation of 10 cm between them. Find μ.

Sol



$$F_c = f_r$$

$$\frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2} = \mu m g$$

$$\mu = \frac{9\times10^9 \times (1\times10^{-7})^2}{\left(\frac{10}{100}\right)^2 \times 9.8 \times \frac{5}{1000}}$$

$$\mu = \frac{9}{5\times9.8} 10^{-14+9+3} = 0.18367$$

Q Suppose the spheres A and B have Identical size and Identical charge. A third sphere 'c' of same size but Uncharged is brought in Contact with the first, then brought in Contact with the second, and finally removed from both. What is the new force of Repulsion between A and B.

Sol

A^{q} B^{q} $C^{q=0}$

q (A)(C) $\rightarrow \frac{q+0}{2} = \frac{q}{2}$ (A) $q/2$ (C) $q/2$.

$q/2$ (C)(B) ^{q} $\rightarrow \frac{q/2+q}{2} = \frac{3q}{4}$. (B) $3q/4$ (C) $3q/4$.

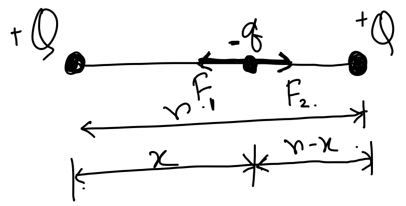
(A) $q/2$ (B) $3q/4$.

$$F = K \frac{q/2 \times 3q/4}{r^2}$$

$$F = K \frac{3q^2}{8r^2}$$

Q Two Identical charges, Q each, are kept at a distance of r from each other. a third charge q is placed on the lining joining the above two charges, such that all the three charges are at equilibrium. What is the magnitude sign and position of the charge q.

Sol



$F_1 = F_2$.

$$\frac{1}{4\pi\varepsilon}\frac{Qq}{x^2} = \frac{1}{4\pi\varepsilon}\frac{Qq}{(r-x)^2}$$

$$x^2 = (r-x)^2$$

$$x = r - x \quad \therefore \quad 2x = r.$$

$$x = \frac{r}{2}.$$

Mid point of r. Ans

$$F = \frac{1}{4\pi\varepsilon}\frac{Qq}{r/2} = \frac{1}{2\pi\varepsilon}\frac{Qq}{r} \rightarrow \text{Mag}$$

Ans

Sign $\rightarrow$ –ve.

Position $\rightarrow$ Midpoint.

Q A charge Q is divided into two objects. What should be the Value of charges on the two objects so that the force between the objects be maximum?

Sol Let one of the object have charge q then the other is $(Q-q)$.

$$F = K\frac{q(Q-q)}{r^2}.$$

To find max we need to differenciate and make equle to zero.

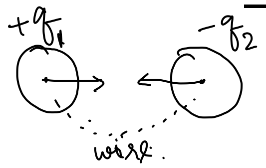
$$\frac{dF}{dq} = \frac{K}{r^2}\frac{d}{dq}(Qq - q^2).$$

$$\left.\frac{dF}{dq}\right|_{max} = \frac{K}{r^2}(Q - 2q) = 0. \qquad Q = 2q \quad \therefore \quad q = \frac{Q}{2}.$$

$\therefore$ charge needs to be divided equally.

Q To Identical sphere, having charge of opposite sign attract each other with a force of 0.108N when separated by 0.5m. spheres are connected by conducting wire, which then removed and there after they repel each other with a force of 0.036N. What were the Initial charges on the sphere?

Sol



$q = \frac{q_1 - q_2}{2}$

$F_A = K\frac{q_1 q_2}{r^2} = K\frac{q_1 q_2}{(.5)^2} = 0.108 \rightarrow q_1 q_2 = \frac{.108 \times .25}{K}$

$= \frac{\cancel{0.108}^{.12} \times 0.25}{\cancel{9} \times 10^9}$

$= 3 \times 10^{-3} \times 10^{-9}$

$F_R = K\frac{\left(\frac{q_1 - q_2}{2}\right)\left(\frac{q_1 - q_2}{2}\right)}{(.5)^2} = 0.036$

$(q_1 - q_2)^2 = \frac{0.036 \times .25 \times 4}{K}$

$= \frac{0.036 \times 0.25 \times 4}{9 \times 10^9}$

$= 4 \times 10^{-3} \times 10^{-9}$

$= 4 \times 10^{-12}$

$q_1 q_2 = 3 \times 10^{-12}$ —— (1)

$q_1 - q_2 = 2 \times 10^{-6}$ —— (2)

$(2 \times 10^{-6})^2 = (q_1 + q_2)^2 - 4 \times 3 \times 10^{-12}$

$4 \times 10^{-12} = (q_1 + q_2)^2 - 12 \times 10^{-12}$

$(q_1 + q_2)^2 = +16 \times 10^{-12}$

$(a-b)^2 = a^2 + b^2 - 2ab + 2ab - 2ab$

$(a-b)^2 = (a+b)^2 - 4ab$

$q_1 + \cancel{q_2} = 4 \times 10^{-6}$

$q_1 - \cancel{q_2} = 2 \times 10^{-6}$

$\cancel{2}q_1 = \cancel{6}^{3} \times 10^{-6}$

$q_1 = 3 \times 10^{-6}$ C

$q_1 - q_2 = 2 \times 10^{-6}$

$3 \times 10^{-6} - q_2 = 2 \times 10^{-6}$

$q_2 = 1 \times 10^{-6}$ C

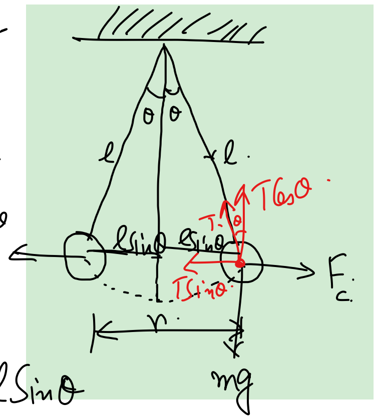
Q Two small sphere each having mass 'm' Kg and charge q are suspended from a point by insulating thread each l meters long. If θ is the angle, each tread makes with the vertical when equilibrium has been attended show that

$$q^2 = (4mgl^2 \sin^2\theta \tan\theta)\, 4\pi\varepsilon_0$$

Sol

$\sin\theta = \frac{P}{l}$

$P = l\sin\theta$



$r = 2l\sin\theta$

$$\frac{mg = \cancel{T}\cos\theta}{F_c = \cancel{T}\sin\theta} = \frac{mg}{F_c} = \frac{1}{\tan\theta}$$

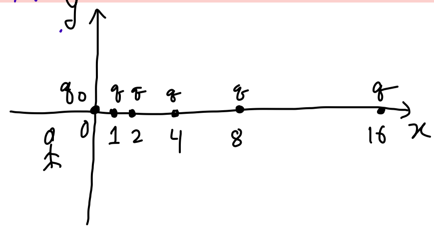
$mg\tan\theta = \frac{1}{4\pi\varepsilon}\frac{q^2}{r^2}$

$q^2 = 4\pi\varepsilon\, mg\tan\theta \times (2l\sin\theta)^2$

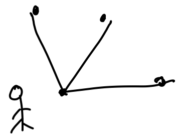
$q^2 = 4\pi\varepsilon_0 (4mgl^2\sin^2\theta\tan\theta)$

Q An infinite number of charges each equal to $4\mu C$ are placed along x axis at $x = 1m$, $x = 2m$, $x = 4m$, $x = 8m$ and so on. Find the total force on the charge of $1C$, placed at the origin.

Sol



$q_0 = 1C$; $q = 4\mu C$.



$$F = \frac{q q_0}{4\pi\varepsilon r^2}$$

$$= \frac{q_0}{4\pi\varepsilon}\left[\frac{q}{1^2} + \frac{q}{2^2} + \frac{q}{4^2} + \cdots\cdots\right]$$

$$F = \frac{q_0 q}{4\pi\varepsilon}\underbrace{\left[1 + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \cdots\right]}_{G.P. \quad \frac{a}{1-r}}$$

$r = \frac{1/2^2}{1} = \frac{1}{4}$

$r = \frac{1/4^2}{1/2^2} = \frac{1}{4}$

$r = \frac{1/8^2}{1/4^2} = \frac{1}{4}$

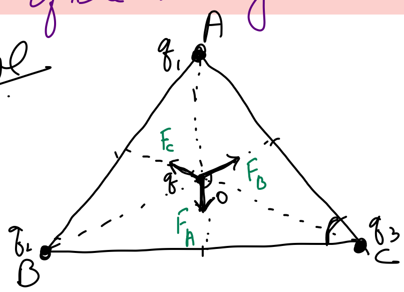
$$F = 1 \times 4 \times 9 \times 10^9\left[\frac{1}{1 - \frac{1}{4}}\right] \times 10^{-6}$$

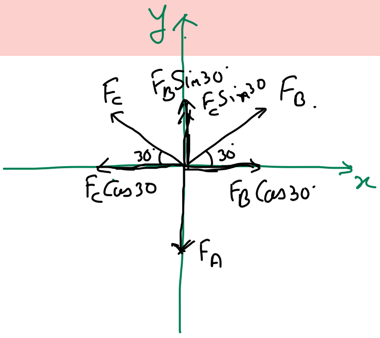
$$= 36 \times 10^9\left[\frac{4}{4-1}\right]10^{-6} = \cancel{36}^{12} \times 10^9 \times \frac{4}{\cancel{3}} \times 10^{-6}$$

$$= 48 \times 10^9 \times 10^{-6} = 48 \times 10^3 \ N. \ Ans$$

Q Consider 3 charges q_1 q_2 q_3 each equal to q at the vertices of equilateral triangle of side l. What is the force on the charge Q placed at the centroid of the triangle.

Sol





$$\left.\begin{aligned} F_A &= \frac{1}{4\pi\varepsilon}\frac{q_1 q}{r} \\ F_B &= \frac{1}{4\pi\varepsilon}\frac{q_1 q}{r} \\ F_C &= \frac{1}{4\pi\varepsilon}\frac{q_1 q}{r} \end{aligned}\right\} \quad F_A = F_B = F_C = F$$

y axis.

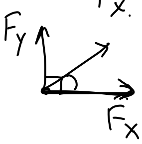
$$F_y = F_B \sin\theta + F_C \sin\theta - F_A$$

$$F_y = 2F\sin\theta - F$$

x axis

$$F_x = F_B \cancel{\cos 30} - F_C \cancel{\cos\theta}$$

$$F_x = 0$$



$$F_R = \sqrt{F_x^2 + F_y^2 + \cancel{2F_x F_y \cos\theta}}$$

$\theta = 90°$

$$F_R = \sqrt{F_x^2 + F_y^2}$$

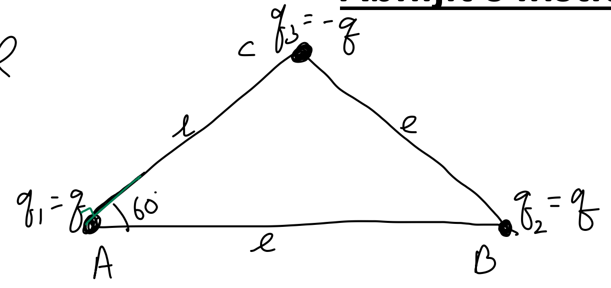
$$F_R = \sqrt{0 + (2F\sin 30 - F)^2}$$

$$F_R = \cancel{2}F \times \frac{1}{\cancel{2}} - F = F - F$$

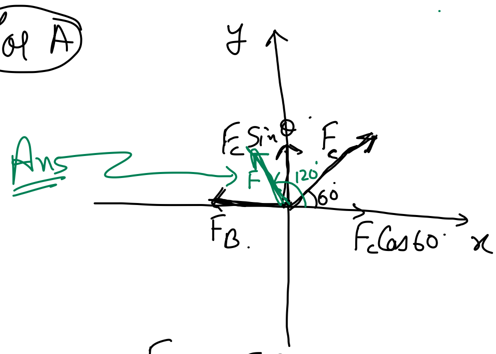
$$\boxed{F_R = 0N}$$

Q For equilateral triangle, $q, q, -q$ is placed at vertices as shown. What is the force on each charge.

Sol



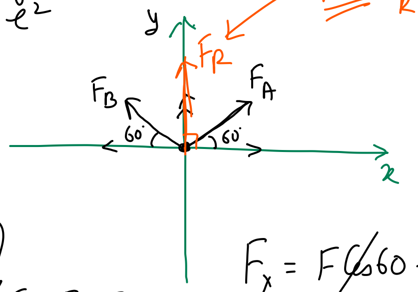
For A



$F_C = K\dfrac{q^2}{l^2}$ $\quad F_A = F_B = F_C = F$.

$F_B = K\dfrac{q^2}{l^2}$

For (C)



Ans $F_R = \sqrt{3}F \angle \pi/2$

$F_B = K\dfrac{q^2}{l}$

$F_A = K\dfrac{q^2}{l}$

$\Big\}$ $F_A = F_B = F$.

$F_x = F\cancel{\cos 60} - F\cancel{\cos 60}$

$= 0$

$F_y = 2F\sin 60$

$= \cancel{2}F \times \dfrac{\sqrt{3}}{\cancel{2}} = \sqrt{3}F$.

$\tan\theta = \dfrac{F_y}{F_x}$

$= \dfrac{\sqrt{3}F}{0}$

$\tan\theta = \infty$

$\theta = 90°$

$F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{0^2 + (\sqrt{3}F)^2}$

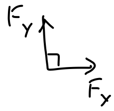
$F_R = \sqrt{3}F$.

$F_x = F\cos\theta - F = F\cos 60 - F$.

$= F\left[\dfrac{1}{2} - 1\right] = -\dfrac{F}{2}$.

$F_y = F\sin\theta$.

$= F\sin 60 = \dfrac{\sqrt{3}}{2}F$.

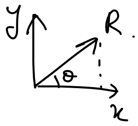


$F = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{\left(\dfrac{F}{2}\right)^2 + \left(\dfrac{\sqrt{3}}{2}F\right)^2}$

$= F$

$$\boxed{F = \frac{1}{4\pi\varepsilon}\frac{q^2}{l^2}}$$



$\tan\theta = \dfrac{y}{x}$.

$\tan\theta = \dfrac{F_y}{F_x}$

Sin	All
Tan	Cos.

$= \dfrac{\sqrt{3}/2\, F}{-F/2}$.

$\tan\theta = -\sqrt{3}$.

$\tan\theta = \tan 120°$

$\theta = 120°$

Q An electron moves a distance of 6cm when accelerated from rest by electric field of strength 2×10^{-4} N/C. Calculate the time of travel. $m_e = 9\times10^{-31}$ kg.

Sol

$x = 6cm$.

$E = 2\times10^{-4}$ N/C.

$t = ??$

$F = qE = ma$.

$a = \dfrac{qE}{m} = \dfrac{1.6\times10^{-19} \times 2\times10^{-4}}{9\times10^{-31}}$

$= \dfrac{3.2}{9} \times 10^{-19-4+31}$

$= 0.355\times10^{8}\ m/s^2$.

$x = ut + \dfrac{1}{2}at^2$.

$\dfrac{6}{100} = 0 + \dfrac{1}{2} 0.355\, t^2 \times 10^{8}$

$t = \sqrt{\dfrac{12}{.355\times10^{10}}} = 5.814\times10^{-5}$ Seconds.

Q An electron falls through a distance of 1.5 cm in $E = 2\times10^4$ N/C. (Fig a) The direction of field is reversed and a proton falls through same distance (Fig b). Compute the time of fall in each case.

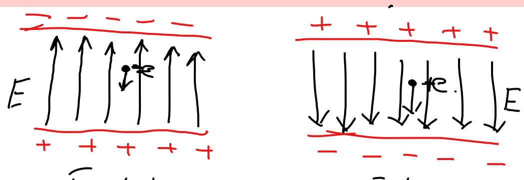


Fig (a) Fig (b)

Sol

$F = ma$

$qE = ma$

$$\boxed{a = \frac{qE}{m}}$$

Fig a

$$a_e = \frac{qE}{m} = \frac{1.6\times10^{-19}\times2\times10^4}{9\times10^{-31}}$$

$x = 1.5\times10^{-2}$ m

$$x = \cancel{ut} + \frac{1}{2}at^2 \rightarrow t^2 = \frac{2x}{a};\ t = \sqrt{\frac{2\times1.5\times10^{-2}\times9\times10^{-31}}{1.6\times10^{-19}\times2\times10^4}} =$$

$$= 2.9\times10^{-9}\,s.$$

Fig b

$$a_e = \frac{qE}{m} = \frac{1.6\times10^{-19}\times2\times10^4}{1.67\times10^{-27}}$$

$$x = ut + \frac{1}{2}at^2 \rightarrow t^2 = \frac{2x}{a} = \quad t = \sqrt{\frac{2\times1.5\times10^{-2}\times1.67\times10^{-27}}{1.6\times10^{-19}\times2\times10^4}} = 1.25\times10^{-7}\,s.$$

Q A charge particle of charge 2 μC and mass 10 mg moving with a velocity of 1000 m/s enters a uniform electric field of strength 10^3 N/C directed perpendicular to the direction of motion. Find the velocity and displacement of the particle after 10 seconds.

Sol

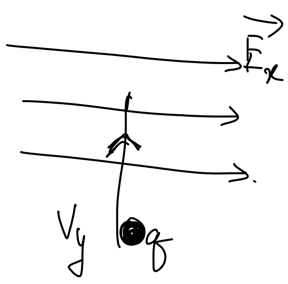
$q = 2\,\mu C$

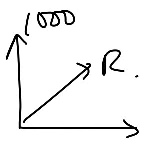
$m = 10\times10^{-6}$ kg

$V = 1000$ m/s

$E = 10^3$ N/C

$V = ??$ $x = ??$ $t = 10$ sec





$V_y = 1000$ m/s

$E_x = 10^3$ N/C

$F_x = qE_x = ma_x$

$a_x = \frac{qE_x}{m}$

$V_x = u_x + a_x t$

$V_x = 0 + \frac{qE_x}{m}\cdot t$

$$= \frac{2\times\cancel{10^{-6}}\times10^3\times\cancel{10}}{\cancel{10}\times\cancel{10^{-6}}} = 2\times10^3$$

$V_x = 2000$ m/s

$$V = R = \sqrt{1000^2 + 2000^2}$$

$$= \sqrt{10^6 + 4\times10^6}$$

$$V = R = 10^3\sqrt{5}\ \text{m/s}$$

$V_y = 1000$ m/s. $v = \frac{y}{t}$

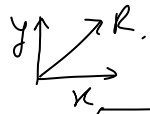
displacement $y = 1000\times10 = 10{,}000$ m.

$x = u_x t + \frac{1}{2}a_x t^2$

$= 0 + \frac{1}{2}\frac{qE_x}{m}t^2$

$$x = \frac{1}{2}\,\frac{2\times10^{-6}\times10^3}{10\times10^{-6}}\times10^2$$

$x = 10000$ m.



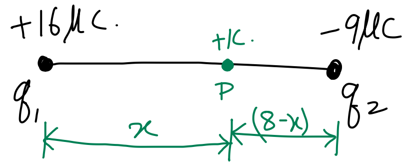
$R = \sqrt{x^2 + y^2}$.

$D = R = \sqrt{(10000)^2 + (10000)^2}$

$= 10000\sqrt{2}$. m **Ans.**

Q Two Point charge $+16\mu C$ and $-9\mu C$ are placed 8 cm apart in air. Determine the position of the point at which the resultant field is zero.

Sol



At Point P:

$E_1 + E_2 = 0$

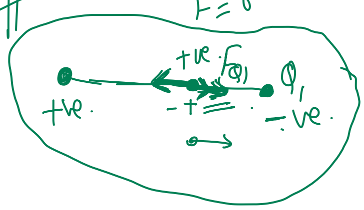
$$\cancel{\frac{1}{4\pi\varepsilon}} \cdot \frac{16\mu C}{x^2} + \cancel{\frac{1}{4\pi\varepsilon}} \frac{9\mu C}{(8-x)^2} = 0$$

$$\frac{4}{x} = \frac{3}{8-x} \Rightarrow 32 - 4x = 3x.$$

$32 = 7x$.

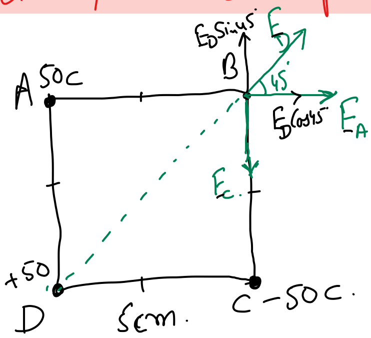
$x = \frac{32}{7} = 4.57$ cm.

#



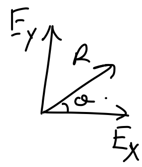
Q ABCD is a square of Side 5 cm. charges of $+50C$, $-50C$ and $+50C$ are placed at A, C and D resp. Find the resultant field at B.

Sol



$$E_x = E_A + E_D \cos 45 = 18\times10^{13} + \frac{9}{\sqrt{2}}\times10^{13} = \left(\frac{18\sqrt{2}+9}{\sqrt{2}}\right)10^{13}\ N/C$$

$$E_y = E_D \sin 45 - E_C = \frac{9}{\sqrt{2}}\times10^{13} - 18\times10^{13} = \left(\frac{9-18\sqrt{2}}{\sqrt{2}}\right)10^{13}\ N/C.$$



$$E_R = \sqrt{E_x^2 + E_y^2}.$$

$$= \sqrt{\left(\frac{9+18\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{9-18\sqrt{2}}{\sqrt{2}}\right)^2} \times 10^{13}$$

$$= \sqrt{\frac{1}{2}\left[(9+18\sqrt{2})^2 + (9-18\sqrt{2})^2\right]} \times 10^{13}$$

$$= \sqrt{\frac{1}{2}\left[81 + 648 + 81 + 648\right]} \times 10^{13}$$

$$= \sqrt{\frac{1458}{2}} \times 10^{13}$$

$$E_A = \frac{1}{4\pi\varepsilon} \cdot \frac{50}{(5\times10^{-2})^2} = \frac{9\times10^9 \times \cancel{50}^{\,2}}{\cancel{25}\times10^{-4}}$$

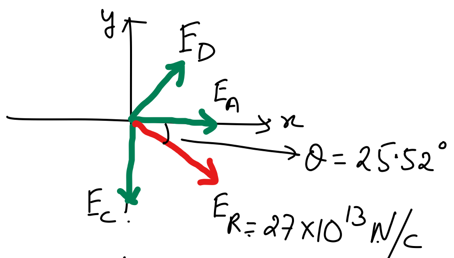
$= 18\times10^{13}$ N/C.

$$E_D = \frac{1}{4\pi\varepsilon} \cdot \frac{50}{(5\sqrt{2}\times10^{-2})^2} = \frac{9\times10^9\times\cancel{50}}{\cancel{50}\times10^{-4}}$$

$= 9\times10^{13}$ N/C.

$$E_C = \frac{1}{4\pi\varepsilon} \frac{50}{(5\times10^{-2})^2} = 18\times10^{13}\ N/C.$$

$E_R = 27\times10^{13}$ N/C. **A**



Ans

$$\tan\theta = \frac{E_y}{E_x} = \frac{\frac{9-18\sqrt{2}}{\sqrt{2}}\times10^{13}}{\frac{9+18\sqrt{2}}{\sqrt{2}}\times10^{13}} = \frac{1-2\sqrt{2}}{1+2\sqrt{2}} = \frac{-1.828}{3.828}$$

$\tan\theta = -0.4775$.

$\theta = \tan^{-1}(-0.4775) = -25.52°$.

Q A charge sphere has surface charge density $\sigma = 0.7\ C/m^2$. If its charge is increased by $0.44C$, the charge density changes by $0.14 C/m^2$. Find the radius of the sphere and Initial charge on it.

Sol –

$$\sigma = \frac{q}{A} = \frac{q}{4\pi R^2} = 0.7$$

Case II

$$\frac{q+0.44}{4\pi R^2} = 0.14 + 0.7 = 0.84$$

$$\frac{q}{0.7} = \frac{q+0.44}{0.84} \longrightarrow q = 2.2\ C$$

$$\frac{2.2}{4\pi R^2} = 0.7 \longrightarrow R = \sqrt{\frac{2.2}{4\pi \times 0.7}} = 0.5\ m$$

Q 64 drops of Radius 0.02 m and each carrying a charge of $5\mu C$ are combined to form a bigger drop. Find How the surface density of electrification will change if no charge is lost.

Sol Small drop

$r = 0.02$

$q = 5\mu C$

$$V = \frac{4}{3}\pi r^3$$

$$\text{Total } V = 64 \times \frac{4}{3}\pi (0.02)^3$$

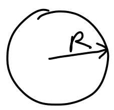
charge

$q = 5\mu C$

64 drop

$q = 5 \times 64 \mu C$

large drop =



$$V = \frac{4}{3}\pi R^3$$

$$64 \times \cancel{\frac{4}{3}}\cancel{\pi}(0.02)^3 = \cancel{\frac{4}{3}}\cancel{\pi} R^3$$

$$4 \times 0.02 = R = 0.08\ m$$

Small drop

$$\sigma_S = \frac{q}{A} = \frac{5\mu}{4\pi r^2}$$

large drop –

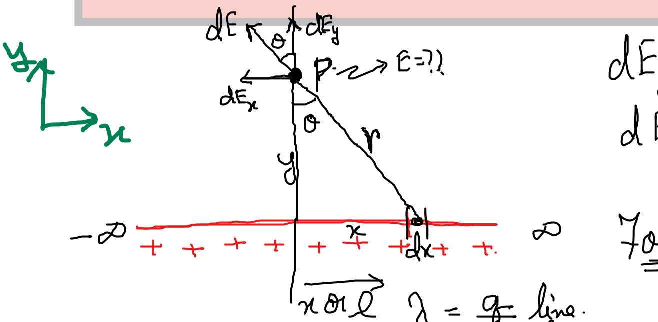
$$\sigma_L = \frac{5 \times 64 \mu}{4\pi R^2}$$

$$\frac{\sigma_S}{\sigma_L} = \frac{\cancel{5\mu}/\cancel{4\pi} r^2}{\cancel{5} \times 64 \cancel{\mu}/\cancel{4\pi} R^2} = \frac{R^2}{64 r^2} = \frac{\overset{4}{\cancel{0.08}} \times \overset{4}{\cancel{0.08}}}{\underset{4}{\cancel{\underset{16}{\cancel{64}}}} \times \cancel{0.02} \times \cancel{0.02}}$$

$$\frac{\sigma_S}{\sigma_L} = \frac{1}{4} \longrightarrow \sigma_L = 4\sigma_S$$

– The large drop will have 4 times more surface charge density than the small drop.

Electric field of a line charge.



$$dE_y = dE\cos\theta \quad \text{——— (1)}$$

$$dE_x = -dE\sin\theta \quad \text{——— (2)}$$

For y Component.

$\lambda = \frac{q}{l}$ line charge density.

$$E = \int dE_y = \int_{-\infty}^{\infty} \cos\theta \, dE$$

$$E = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}$$

$q = \lambda l$

$dq = \lambda dx$

$$dE = \frac{1}{4\pi\varepsilon}\frac{dq}{r^2}$$

$$= \frac{1}{4\pi r}\frac{\lambda dl}{r^2}$$

$r = \sqrt{x^2+y^2}$

$$dE = \frac{1}{4\pi r}\frac{\lambda dx}{(x^2+y^2)}$$

$$E = \int_{-\infty}^{\infty} \cos\theta \cdot \frac{1}{4\pi\varepsilon}\frac{\lambda dx}{(x^2+y^2)}$$

$$= 2\int_0^{\infty} \frac{1}{4\pi\varepsilon}\frac{\cos\theta \, \lambda dx}{(x^2+y^2)}$$

$$E = \frac{2\lambda}{4\pi\varepsilon}\int_{x=0}^{\infty} \frac{\cos\theta \, dx}{(x^2+y^2)}$$

$x = y\tan\theta \longrightarrow \tan\theta = \frac{x}{y} \longrightarrow x=0, \ \tan\theta = 0, \ \theta = 0°$

$x = \infty, \ \tan\theta = \infty, \ \theta = 90° = \pi/2.$

$$\frac{dx}{d\theta} = y\sec^2\theta$$

$$dx = y\sec^2\theta \, d\theta.$$

$$= \frac{\lambda}{2\pi\varepsilon}\int_{\theta=0}^{\pi/2} \frac{\cos\theta \cdot y\sec^2\theta \, d\theta}{(y^2\tan^2\theta + y^2)}$$

$$= \frac{\lambda}{2\pi\varepsilon}\int_{\theta=0}^{\pi/2} \frac{\cos\theta\sec^2\theta \, y}{y^2(1+\tan^2\theta)} d\theta$$

$$= \frac{\lambda}{2\pi\varepsilon}\int_0^{\pi/2} \frac{\cos\theta\sec^2\theta \, \cancel{y}}{y^{\cancel{2}}(1+\tan^2\theta)} d\theta.$$

$$= \frac{\lambda}{2\pi\varepsilon}\int_0^{\pi/2} \frac{\cos\theta \, \cancel{\sec^2\theta}}{y\,\cancel{\sec^2\theta}} d\theta.$$

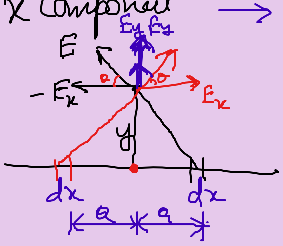
$$= \frac{\lambda}{2\pi\varepsilon y}\Big[\sin\theta\Big]_0^{\pi/2}$$

$$= \frac{\lambda}{2\pi\varepsilon y}\left[\sin\frac{\pi}{2} - \sin 0\right]$$

$$E = \frac{\lambda}{2\pi\varepsilon y}[1-0] = \frac{\lambda}{2\pi\varepsilon y}.$$

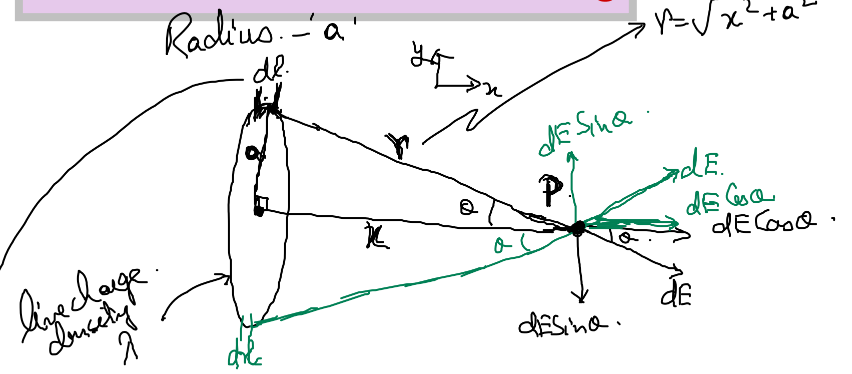
$$\boxed{E = \frac{\lambda}{2\pi\varepsilon y}}$$

For x Component → every charge element on the right has a corresponding charge element on the left. They are equal and opposite ∴ the cancel.



No effect of x axis component on electric field E.

Electric field over a ring.



$\lambda = \frac{q}{l}$

$q = \lambda l$

$dq = \lambda dl$

$E = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}$

As the total charge q is uniformly distributed, the charge dq on the element dl is

$dq = \left(\frac{q}{2\pi a}\right) dl$

$dE = \frac{1}{4\pi\varepsilon}\frac{dq}{r^2}$

$= \frac{1}{4\pi\varepsilon}\cdot\left(\frac{q}{2\pi a}\right)\frac{dl}{r^2}$

$E = \int_0^{2\pi a} dE\cos\theta$ → only axials component because perpendicular component cancels out

$\cos\theta = \frac{x}{r}$

$E = \int_0^{2\pi a} \frac{x}{r}\, dE$

$E = \int_0^{2\pi a} \frac{x}{r}\cdot\underbrace{\left(\frac{1}{4\pi\varepsilon}\right)}_{K}\cdot\left(\frac{q}{2\pi a}\right)\frac{1}{r^2}\cdot dl$

$E = \frac{Kqx}{2\pi a r^3}\int_0^{2\pi a} dl$

$= \frac{Kqx}{2\pi a r^3}\left[l\right]_0^{2\pi a}$

$= \frac{Kqx}{\cancel{2\pi a} r^3}\cdot\left[\cancel{2\pi a} - 0\right]$

$= K\frac{qx}{r^3} = Kqx\frac{1}{\left(\sqrt{x^2+a^2}\right)^3}$

$$\boxed{E = \frac{1}{4\pi\varepsilon}\frac{qx}{(x^2+a^2)^{3/2}}}$$

Case I At Center

$x=0$

$E = \frac{1}{4\pi\varepsilon}\cdot 0 = 0$ → Electric feild at center is always zero.

Case II $x \gg a$

$\therefore \frac{a}{x} \lll 1$

$E = \frac{1}{4\pi\varepsilon}\frac{qx}{\left[x^2\left(1+\frac{a^2}{x^2}\right)\right]^{3/2}} = \frac{qx}{\left[x^2(1+0^2)\right]^{3/2}}\cdot\frac{1}{4\pi\varepsilon}$

$= \frac{qx}{x^{3/2\cdot 2}(1+0)^{3/2}}$

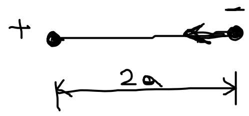
$$\boxed{E = \frac{1}{4\pi\varepsilon}\frac{q}{x^2}}$$

Dipole - Two equal and opposite charges placed very close to each other.
In dipole we analyze - 1. electric field of dipole. 2. Torque in a dipole.

Dipole moment- It measures the strength of electric dipole.

Dipole moment = Charge X Distance from negative to positive charge.

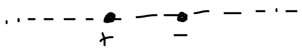
$$\boxed{\vec{p} = q \times 2\vec{a}}$$



Unit – C m. Coulomb meter

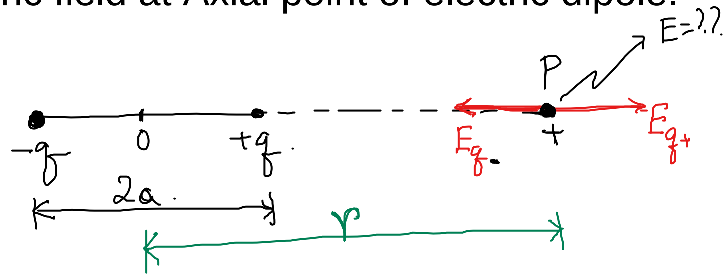
Dipole field- It is the electric field produced by the dipole.

1. Axial Point



2. Equatorial Point

1. Electric field at Axial point of electric dipole.



Total electric field $E_T = E_{q+} + E_{q-}$

$$E_T = \frac{1}{4\pi\varepsilon}\frac{q}{(r-a)^2} + \frac{-q}{4\pi\varepsilon(r+a)^2} = \frac{1}{4\pi\varepsilon} q\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

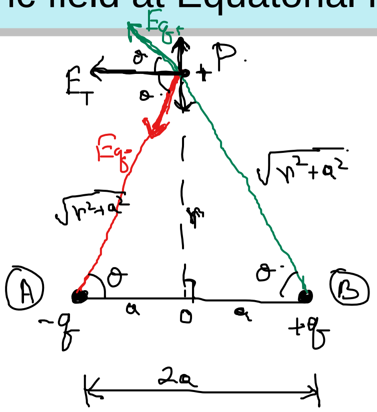
$$= \frac{q}{4\pi\varepsilon}\left[\frac{(r+a)^2-(r-a)^2}{(r-a)^2(r+a)^2}\right] = \frac{q}{4\pi\varepsilon}\frac{\cancel{r^2}+\cancel{a^2}+2ar-\cancel{r^2}-\cancel{a^2}+2ar}{(r^2-a^2)^2}$$

$$= \frac{q}{4\pi\varepsilon}\frac{4ar}{(r^2-a^2)^2} \qquad \boxed{p = q\cdot 2a}$$

$$\boxed{\vec{E} = \frac{2r\vec{p}}{4\pi\varepsilon_0(r^2-a^2)^2}} \longrightarrow \text{from -ve to +ve}$$

If $r \gg a$. $\vec{E} = \frac{r\vec{p}}{2\pi\varepsilon(r^2-0)^2} = \frac{\cancel{r}\vec{p}}{2\pi r^{\cancel{4}3}}$ $\qquad \boxed{\vec{E} = \frac{\vec{p}}{2\pi\varepsilon_0 r^3}}$ from -ve to +ve.

Electric field at Equatorial Point of dipole.



$$\vec{E}_{q+} = \frac{1}{4\pi\varepsilon}\frac{+q}{(\sqrt{r^2+a^2})^2} \quad \text{along } \underline{BP}.$$

$$E_{q-} = \frac{1}{4\pi\varepsilon}\frac{-q}{(\sqrt{r^2+a^2})^2} \quad \text{along } \underline{PA}.$$

Normal Component (y compont)

$E_{q+} = E_{q-}$ — Magnitude will remain same.

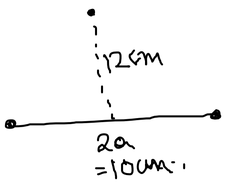
$$\frac{1}{4\pi\varepsilon}\frac{q}{(r^2+a^2)} = \frac{1}{4\pi\varepsilon}\frac{-q}{(r^2+a^2)}$$

$\therefore$ Normal Component cancels out.

\# Calculate the electric feild due to electric dipole of lenght 10cm having charge 1μC at an equatorial point 12cm from the Center of the dipole

Sol

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2qa}{(r^2+a^2)^{3/2}}$$



$a = 5cm = 5\times10^{-2}\,m$.

$$= 9\times10^9 \times \frac{2\cdot 1\times10^{-6}\times 5\times10^{-2}}{\left[(12\times10^{-2})^2 + (5\times10^{-2})^2\right]^{3/2}}$$

$$= 4.096\times10^5\ N/C.$$

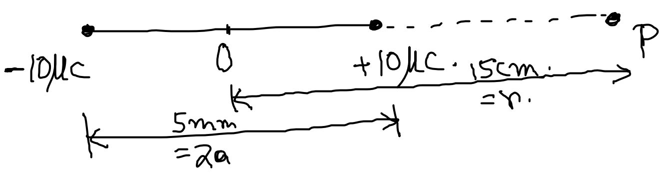
\# Two charges ±10μC are placed 5mm apart. Determine the electric feild at (a) a point P on the axis of the dipole 15cm away from the Center O on the side of the +ve charge (b) A point Q 15cm away from O on the line passing through O and normal to the axis of the dipole.

Sol (1) For axial $E = \frac{1}{4\pi\varepsilon}\frac{2p}{r^3}$ $\quad p = q\cdot 2a =$

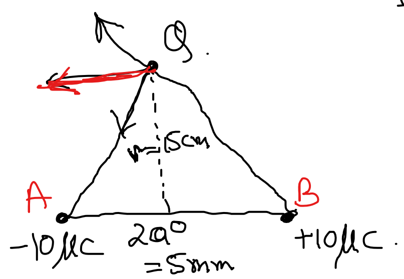
$$= \frac{9\times10^9\times2\times q\times2a}{(15\times10^{-2})^3} = \frac{9\times10^9\times2\times10\times10^{-6}\times5\times10^{-3}}{(15\times10^{-2})^3}$$

$$= 2.66\times10^5\ N/C.$$

Ans



(2)

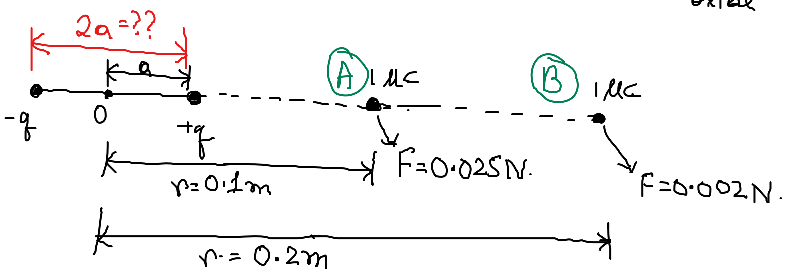


$$\vec{E} = \frac{p}{4\pi\varepsilon r^3} = \frac{q\times 2a}{4\pi\varepsilon r^3} = \frac{10\times10^{-6}\times 9\times10^{9}\times 5\times10^{-3}}{(15\times10^{-2})^3}$$

$$= 1.33\times10^{5}\ N/C$$ along BA

Q The force experienced by a unit charge when placed at a distance of 0.1 m from the middle of an electric dipole on its axial line is 0.025 N and when it is placed at a distance of 0.2 m, the force is reduced to 0.002 N. Calculate the dipole length.

Sol $\frac{F}{q} = E \qquad q = 1C \quad \therefore F = E \rightsquigarrow F = E_{axial} = \frac{1}{4\pi\varepsilon}\cdot\frac{2pr}{(r^2-a^2)^2}$



At (A)

$$0.025 = 9\times10^{9}\times\frac{2p\times0.1}{((.1)^2-a^2)^2}$$

At (B)

$$0.002 = 9\times10^{9}\,\frac{2p\times0.2}{((.2)^2-a^2)^2}$$

$$\frac{(A)}{(B)} = \frac{0.025}{0.002} = \frac{\cancel{9\times10^9}\,\frac{\cancel{2p}\times0.1}{((.1)^2-a^2)^2}}{\cancel{9\times10^9}\,\frac{\cancel{2p}\times0.2}{((.2)^2-a^2)^2}}$$

$$\frac{25}{\cancel{2}} = \frac{(0.04-a^2)^2}{\cancel{2}(0.01-a^2)^2} \Rightarrow 5 = \frac{0.04-a^2}{0.01-a^2}$$

$$0.05 - 5a^2 = 0.04 - a^2$$

$$-4a^2 = -0.01 \Rightarrow a^2 = \frac{0.01}{4} = \frac{1}{400}$$

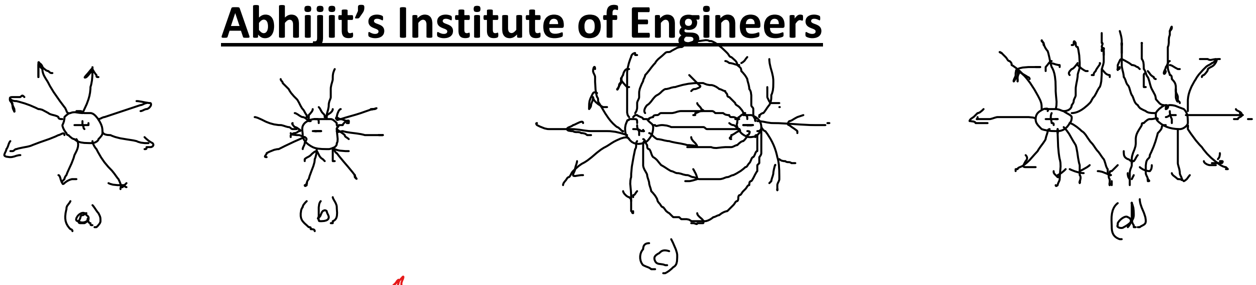
$$a = \sqrt{\frac{1}{400}} = \frac{1}{20} \qquad \boxed{a = 0.05\,m}\ \text{Ans}$$

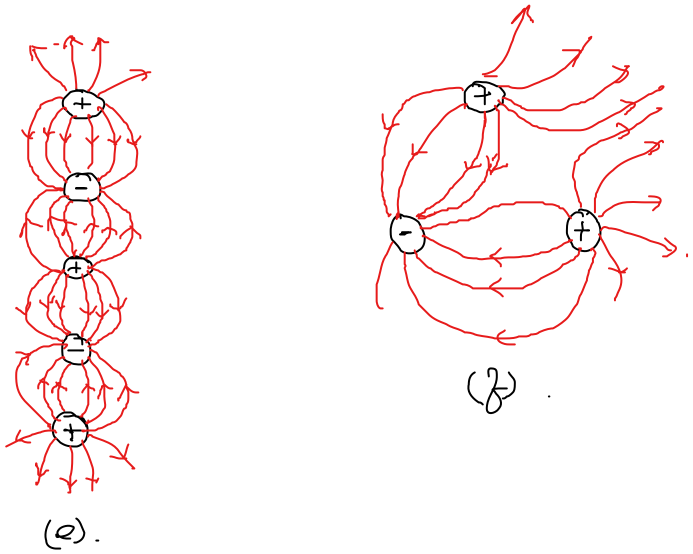
Electric feild lines:-

2016, 2014, 2011, 2012

- It gives the direction of electric feild which is tangent to it.
- It starts from the charge and ends in -ve charge
- These lines never intersect.
- They are continuous.
- They are ⊥ to the surface of charge Conductor.

Feild lines



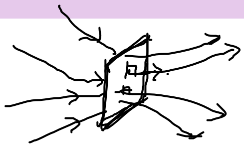


Electric flux. It is the bundle of electric field lines.

→ Scalar.

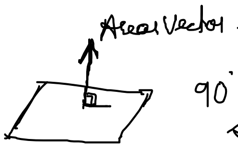
Unit - Wb ⇒ 1 Wb = 10^8 electric field line.

$10^8 = 1$ Wb.



- It is a measure of total number of electric line of forces passing normally through that area.

area → Vector.

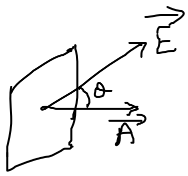


90° with the surface.

$E = \frac{\phi}{A}$ flux per unit area.

$\boxed{\phi = E \cdot A}$

$\boxed{\phi = \vec{E} \cdot \vec{A} = |E| \cdot |A| \cos\theta}$



θ is the angle between $\vec{E}$ and $\vec{A}$.

Normally θ = 0   cos 0 = 1

$\phi = E \cdot A \cos 0°$

$\boxed{\phi = E \cdot A}_{max}$.

In terms of Integration

$\phi = E \cdot A$.

$d\phi = E \cdot dA$

$\boxed{\phi = \int E\, dA}$

1. $0 < \theta < 90°$: φ is +ve.
2. $\theta = 90$: φ is zero.
3. $90° < \theta < 180$: φ is −ve.

Gauss' Theorem

Total flux through a closed surface is $1/\varepsilon_0$ times the net charge enclosed by the closed surface

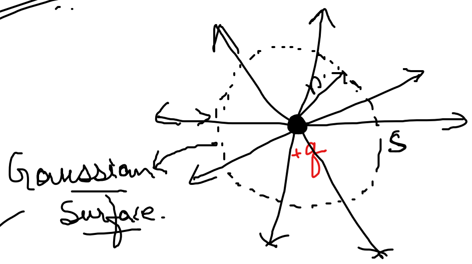
$$\boxed{\phi = \oint \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}} \longrightarrow \text{In air.}$$

$\boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$

$\varepsilon_r = 1$ — Vacuum/Air

- This law is true for any closed surface, no matter what its shape or size
- Often used for symmetric shape

Proof Let's consider a Isolated +ve point charge 'q'.



Electric feild at the surface 's'

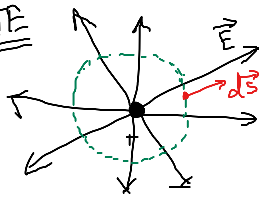
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

$$\phi = \vec{E}\cdot\vec{s} \Rightarrow d\phi = \vec{E}\cdot\vec{ds}$$

$$d\phi = |E|\,|ds|\cos\theta$$

$\theta \rightarrow$ Angle between E and ds

NOTE



Angle between $\vec{E}$ and $\vec{ds}$ $= 0°$.

$\cos 0 = 1$

Any closed path of the charge.

$$d\phi = E\,ds\cos 0$$

$$d\phi = E\,ds.$$

Integrating both sides

$$\int d\phi = E\int ds.$$

ds = Surface Area of Sphere

$$\phi = E\,4\pi r^2.$$

$$= \left(\frac{1}{4\pi\varepsilon}\frac{q}{\cancel{r^2}}\right).\,4\pi\cancel{r^2}$$

$$\boxed{\phi = \frac{q}{\varepsilon}.}$$ proved

Coulomb's law from Gauss Theorem

$$\phi = E\cdot 4\pi r^2$$

Using Gauss theorem $\phi = \frac{q}{\varepsilon}.$

$$\frac{q}{\varepsilon} = E\cdot 4\pi r^2 \Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}.}$$

The force on the point charge q_0 if placed on surface will be.

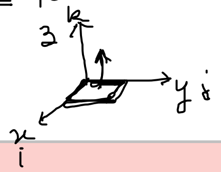
$$F = q_0 E$$

$$\boxed{F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}.} \longrightarrow \text{Coulomb's law.}$$

Q If the electric field is given by $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$ N/C. Find the flux through the surface of area $100\,m^2$ lying in the XY plane.

Sol $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$ N/C.

$S = 100\,m^2$. $\vec{S} = 100\hat{k}$



$\phi = \vec{E} \cdot \vec{S}$

$= (8\hat{i} + 4\hat{j} + 3\hat{k}) \cdot (100\hat{k})$

$\phi = (8\hat{i} + 4\hat{j} + 3\hat{k})(0\hat{i} + 0\hat{j} + 100\hat{k})$

$\phi = 300$ Wb

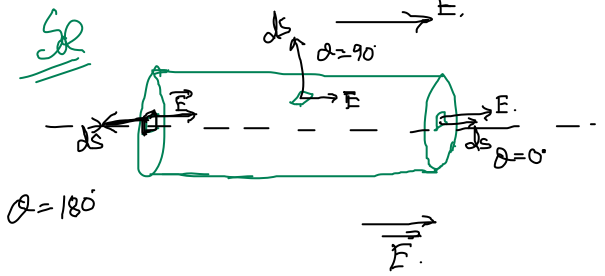
$\hat{i} \cdot \hat{k} = 0$
$\hat{j} \cdot \hat{k} = 0$
$\hat{k} \cdot \hat{k} = 1$

Q The electric field in a certain region of space is $(5\hat{i} + 4\hat{j} - 4\hat{k}) \times 10^5$ N/C. Calculate the flux due to this field over an area of $(2\hat{i} - \hat{j}) \times 10^{-2}\,m^2$.

Sol $\phi = \vec{E} \cdot \vec{S} = 6 \times 10^3$ Wb

Q A Cylinder is placed in a uniform electric field $\vec{E}$ with its axis parallel to the field. Show that the total electric flux through the cylinder is zero.

Sol



$\phi_{Total} = \phi_{Right} + \phi_{left} + \phi_{curve}$.

$\phi_T = \oint_R \vec{E} \cdot \vec{ds} + \oint_L \vec{E} \cdot \vec{ds} + \oint_C \vec{E} \cdot \vec{ds}$

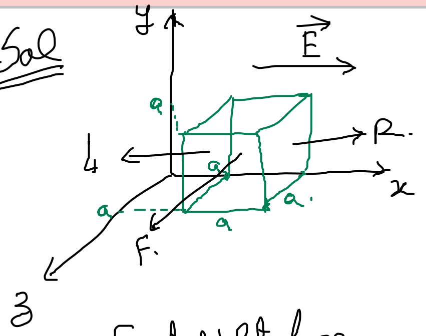
$= \oint E \cdot ds \cdot \cos 0 + \oint E \cdot ds \cdot \cos 180 + \oint_C E \cdot ds \cdot \cos 90$.

$= E\oint ds - E\oint ds + 0 = 0$

Q The electric field Components are shown. $E_x = \alpha x^{1/2}$, $E_y = E_z = 0$. in which $\alpha = 800$ N/Cm². Calculate ① The flux ϕ_E through the Cube. ② The charge within the Cube.

$a = 0.1$ m.

Sol



$E_x = \alpha x^{1/2}$.

E at the left face. $E_L = \alpha \cdot a^{1/2}$.

$\phi = \vec{E} \cdot \vec{ds} = E\, ds \cos\theta$.

$= \alpha a^{1/2} \cdot a^2 \cdot \cos 180$

$\phi_L = -\alpha a^{2+\frac{1}{2}} \Rightarrow -\alpha a^{5/2} = \phi_L$

E at right face. $\phi = 0°$

$E_R = \alpha (2a)^{1/2}$.

$= \sqrt{2}\alpha a^{1/2}$.

$\phi_R = E\, ds \cos\phi$

$= \sqrt{2}\alpha a^{1/2} \times a^2 \times \cos 0 \Rightarrow \phi_R = \sqrt{2}\alpha a^{5/2}$.

Front: $\vec{E}$, $\vec{ds}$, $\cos 90 = 0$

Back: $\vec{ds}$, $\vec{E}$, $\cos 90° = 0$

Top: $\vec{ds}$, $\vec{E}$, $\cos 90 = 0$

Bottom: $\vec{E}$, $\vec{ds}$, $\cos 90 = 0$

$\Big\}$ $\phi = 0$ for all Cases.

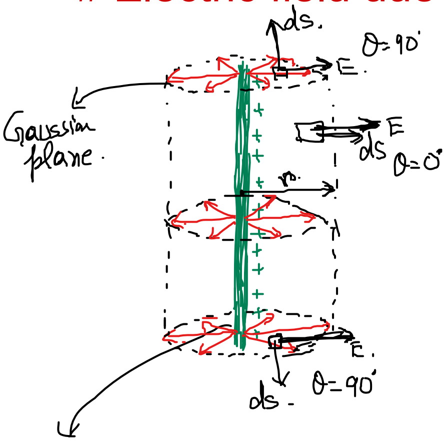
$$\therefore \phi_T = \phi_L + \phi_R = -\alpha a^{5/2} + \sqrt{2}\,\alpha a^{5/2} = \alpha a^{5/2}[\sqrt{2}-1] = 800(.1)^{5/2}[\sqrt{2}-1]$$

$$\Rightarrow 800 \times 3.16\times10^{-3} \times 0.414 = 1.0473\ \text{Wb}\ \text{Ans}$$

(2) $\phi = \dfrac{q}{\varepsilon_0} \Rightarrow q = \phi\,\varepsilon_0$

$$= 1.0473\times8.85\times10^{-12} = 9.269\times10^{-12}\ \text{Wb. Ans}$$

Electric field due to infinitely long charged wire.



$\lambda = \dfrac{q}{l}$ C/m

$q = \lambda l$

$\cos(-\theta) = \cos\theta$

$$\phi = \oint \vec{E}\cdot\vec{ds}$$

$$= \cancel{\int_{top} E\cdot ds\,(\cos 90} + \cancel{\int_{bottom} E\cdot ds\,(\cos(-90)} + \int_{curve} E\cdot ds\cdot\cos 0.$$

$$\phi = E\int_{curve} ds. = E\cdot(2\pi r l) \quad —(1)$$

Curved Surface area of Cylinder.

Gauss' Theorem $\phi = \dfrac{q}{\varepsilon_0} = \dfrac{\lambda l}{\varepsilon_0}$ —(2)

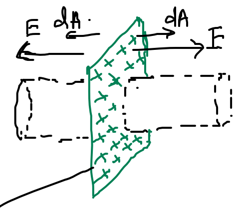
(1) = (2)

$$\frac{\lambda \cancel{l}}{\varepsilon_0} = E\cdot 2\pi r \cancel{l}.$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

electric field due to uniformly charged sheet

Sol



$\rightarrow \sigma = \dfrac{q}{A}$ C/m^2.

$q = \sigma\cdot A$

$$\phi = \oint \vec{E}\cdot\vec{ds} \Rightarrow \phi = \int_{Right} E\,dA\cos 0 + \int_{Left} E\cdot dA\cos(-0).$$

$$\phi = E\int_R dA + E\int_L dA = EA + EA.$$

$$\phi = 2EA \quad —(1)$$

Gauss' Theorem. $\phi = \dfrac{q}{\varepsilon_0} = \dfrac{\sigma\cdot A}{\varepsilon_0}$ —(2)

(1) = (2)

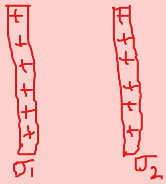
$$2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

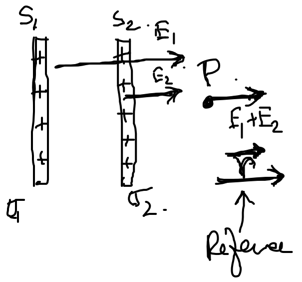
Case I: If the sheet is +vely charged – The field is directed away from sheet.

Case II: If the sheet is −vely charged – " " " " Towards the sheet.

Q. Two Infinite Parallel planes have uniform charge densities of σ_1 and σ_2. Determine the electric field at the point ① left of the sheets ② In between ③ Right of the sheets.



Sol ①



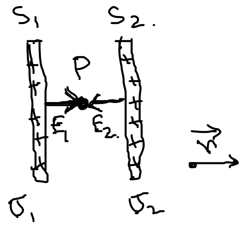
$\vec{E_1}$ & $\vec{E_2}$ are in the same direction.

$$\vec{E_1} = \frac{\sigma_1}{2\varepsilon_0}\hat{r} \quad ; \quad \vec{E_2} = \frac{\sigma_2}{2\varepsilon_0}\hat{r}$$

Resultant $\vec{E} = \vec{E_1} + \vec{E_2}$

$$= \frac{\sigma_1\hat{r}}{2\varepsilon_0} + \frac{\sigma_2\hat{r}}{2\varepsilon_0} = \frac{1}{2\varepsilon_0}(\sigma_1+\sigma_2)\hat{r}$$ Ans

②



Now, $\vec{E_1}$ & $\vec{E_2}$ are in the opp direction.

$$\vec{E_1} = \frac{\sigma_1}{2\varepsilon_0}\hat{r} \quad ; \quad E_2 = \frac{\sigma_2}{2\varepsilon_0}(-\hat{r})$$

Resultant $\vec{E} = \vec{E_1} + \vec{E_2}$

$$= \frac{\sigma_1}{2\varepsilon_0}\hat{r} + \frac{\sigma_2}{2\varepsilon_0}(-\hat{r}).$$

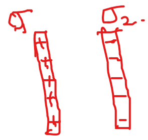
$$\vec{E} = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2)\hat{r}$$ Ans

③

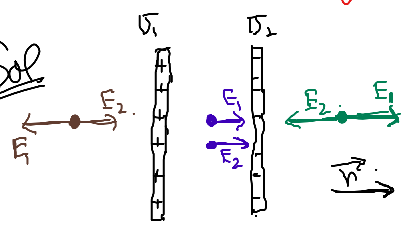
$$\vec{E} = \frac{\sigma_1}{2\varepsilon_0}(-\hat{r}) + \frac{\sigma_2}{2\varepsilon_0}(-\hat{r}).$$

$$\vec{E} = -\frac{1}{2\varepsilon_0}(\sigma_1+\sigma_2)\hat{r}$$ Ans

Q. Repeat the above question for the fig shown



Sol



$$E_1 = \frac{\sigma_1}{2\varepsilon_0}$$

$$E_2 = \frac{\sigma_2}{2\varepsilon_0}.$$

① For Right

$$\vec{E} = E_1 + E_2$$

$$= \frac{\sigma_1}{2\varepsilon_0}\hat{r} + \frac{\sigma_2}{2\varepsilon_0}(-\hat{r})$$

$$= \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2)\hat{r}$$ Ans

② In between.

$$\vec{E} = \frac{\sigma_1}{2\varepsilon_0}\hat{r} + \frac{\sigma_2}{2\varepsilon_0}\hat{r}$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{r}$$ Ans

③ For left

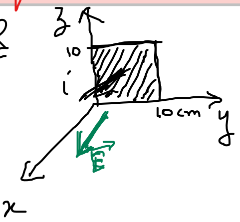
$$\vec{E} = E_1 + E_2$$

$$= \frac{\sigma_1}{2\varepsilon_0}(-\hat{r}) + \frac{\sigma_2}{2\varepsilon_0}(\hat{r}).$$

$$\vec{E} = \frac{1}{2\varepsilon_0}(\sigma_2 - \sigma_1)\hat{r}$$ Ans

Q. If $\vec{E} = 3\times10^3\hat{i}$ N/C ① What is the flux of the field through a square of 10 cm on a side whose plane is parallel to the YZ plane? ② What is the flux through the same square if the normal to its plane makes 60° angle with the x axis?

Sol



$\vec{ds} = (0.1 \times 0.1)\hat{i} = 0.01\hat{i}$

$\vec{E} = 3\times10^3\,\hat{i}$

$\phi = \vec{E}\cdot\vec{ds} \Rightarrow 3\times10^3\,\hat{i} \cdot 0.01\,\hat{i} = 3000 \times \frac{0.01}{100} = 30\,Wb.$

Or

$\phi = E \cdot ds \cdot \cos\theta$

$= 3000 \times .01 \times \cos 0$

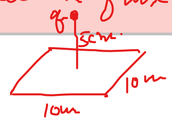
$= 30\,Wb.$

(2) $\theta = 60 \Rightarrow \cos 60 = \frac{1}{2}$.

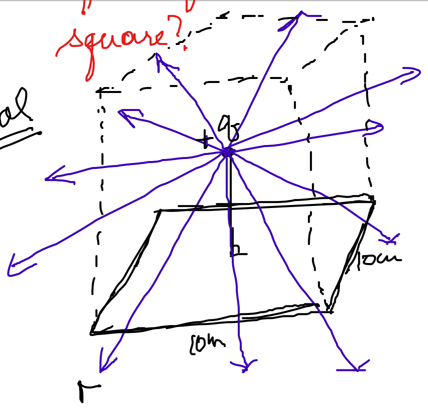
$\vec{E} = \vec{E}\cdot\vec{ds} = 3000 \times .01 \times \cos 60 = 30 \times \frac{1}{2} = 15\,Wb$

Q A point charge of $+10\,\mu C$ is a distance 5cm directly above the surface of the square of side 10cm. as shown. What is the magnitude of electric flux through the square?

Set E 2018



Sol



Symmetric of six planes of a cube about its center ensures that the flux ϕ_s through plane is same.

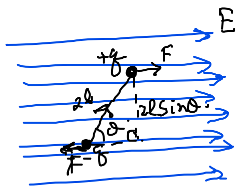
This is possible only when the charge is placed at center.

$\phi_T = 6\phi_s = \frac{q}{\varepsilon_0}$ — Gauss Theorem.

$\phi_s = \frac{q}{6\varepsilon_0} = \frac{1}{6}\,\frac{10\times10^{-6}}{8.85\times10^{-12}}$.

$= 1.88 \times 10^5\,Wb.$

Dipole in a uniform External field



Torque $\tau = F \times$ perpendicular distance.

$\perp$ distance $= 2l\,\sin\theta$.

$F = qE$.

$\tau = qE \cdot 2l\sin\theta$

$\tau = (q \cdot 2l)\,E\sin\theta$.

$\tau = p \cdot E\sin\theta$ $\quad \boxed{p = q \cdot 2l}$

$\boxed{\vec{\tau} = \vec{p}\times\vec{E}}$

Case I $\quad \theta = 0 \rightarrow$ stable equilibrium.

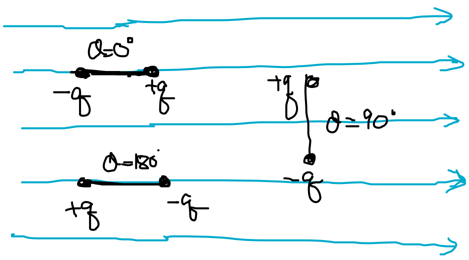
$\tau = p \cdot E\sin 0 = 0$. min

Case II $\quad \theta = 90°$.

$\tau = p \cdot E\sin 90 = p E$. max

Case III $\quad \theta = 180° \rightarrow$ Unstable equilibrium.

$\tau = p \cdot E \sin 180 = 0$.



Q An electric dipole consist of two charges of 0.1 μC separated by a distance of 2 cm. The dipole is placed in an external field of 10^5 N/C. What maximum torque does the field exert on the dipole?

Sol $2l = 2\times10^{-2}$ m.

For Max $\sin\theta = 1$.
$\theta = 90°$

$\tau = \vec{p}\times\vec{E}$
$= p\cdot E\sin\theta$
$= q\cdot 2l\cdot E\cdot 1$.
$= 0.1\times10^{-6}\times 2\times10^{-2}\times10^{5} = 0.2\times10^{-3}$ N·m
$\tau = 2\times10^{-4}$ N·m.

work done on a dipole in a uniform electric field

When an electric dipole is placed in a uniform electric field, it experiences torque and tends to align it in such a way to attend a stable equilibrium.

we know $W = F\cdot x$ — linear.
For Rotation → $W = \tau\theta$.
For small change $dW = \tau d\theta$.
$dW = [p\cdot E\sin\theta]\, d\theta$.

Integrating both sides $\int dW = pE\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta$

$W = pE\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$

$W = pE[\cos\theta_1 - \cos\theta_2] \longrightarrow$ P.E.

∴ Workdone is potential energy (U), in rotating the dipole from angle θ_1 to θ_2.

$$\boxed{U = W = pE[\cos\theta_1 - \cos\theta_2]}$$

We assume the dipole is initially perpendicular to the direction of the field and brought to a condition making an angle of Theta. (θ)

Condition making an angle of θ.

$U = pE[\cos 90 - \cos\theta] = -pE\cos\theta$.

$$\boxed{U = -\vec{p}\cdot\vec{E}}$$

Q. An electric dipole of moment 2×10^{-8} C·m is aligned in a Uniform electric field of 2×10^{4} N/C. Calculate the workdone in rotating the dipole from 30° to 60°.

Sol. $p = 2\times10^{-8}$ C·m.

$E = 2\times10^{4}$ N/C.

$$U = pE[\cos\theta_1 - \cos\theta_2]$$
$$= 2\times10^{-8}\cdot 2\times10^{4}[\cos 30^\circ - \cos 60^\circ]$$
$$= 4\times10^{-4}\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right]$$
$$U = 2(\sqrt{3}-1)\times10^{-4}\ J.$$
$$U = 1.464\times10^{-4} J.$$

2014

Q. An electric dipole of lengt 2 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $8\sqrt{3}$ N·m. Calculate the P.E of dipole, if it has a charge of ± 4 nC.

Sol. $2a = 2\times10^{-2}$ m.

$\theta = 60^\circ$

$\tau = 8\sqrt{3}$ N·m.

$q = \pm 4\times10^{-9}$ C.

$p = q\cdot 2a = 4\times10^{-9}\times 2\times10^{-2}$.

$= 8\times10^{-11}$ cm.

$$U = pE[\cos 90 - \cos 60^\circ]$$
$$U = 8\times10^{-11}\times 2\times10^{11}\left[0 - \frac{1}{2}\right].$$
$$U = -16\times\frac{1}{2} = -8\ J.$$

$$\tau = p\cdot E\sin\theta$$
$$E = \frac{\tau}{p\sin\theta}$$
$$= \frac{8\sqrt{3}}{8\times10^{-11}\times\sin 60^\circ}$$
$$= \frac{\cancel{8}\cancel{\sqrt{3}}\times 2}{\cancel{8}\times10^{-11}\times\cancel{\sqrt{3}}}$$
$$E = 2\times10^{11}\ N/C.$$

2012

Q. Distinguish between dielectric and Conductor –

Sol. Dielectric are non conductors and do not have free electrons at all, while conductors have free electrons in any volume which makes them to pass the electricity through them.

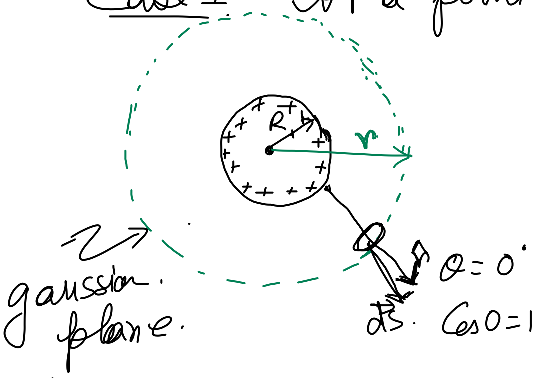
Q. Difference between charge and Mass.

Charge	Mass
1. May be +ve, –ve, or zero.	1. Always +ve.
2. Quantized.	2. yet to be established.
3. Electric charge is Conserved.	3. Mass is not, it is changed to energy.
4. Force of Attraction or Repulsion exists.	4. The gravitational force between two masses is always attractive.
5. $F = Kq_1q_2/r^2$	5. $F = G\, m_1m_2/r^2$

Field due to a uniformly charged thin spherical shell

Let σ be the uniform surface charge density of a thin spherical shell of radius "R". Let find 'E'.

Case I At a point outside the shell. $r > R$.



$$\phi = \int \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$$

$$\phi = \int E \cdot ds \cdot \cos\theta$$

$$\phi = E \int ds.$$

$$\phi = E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{4\pi \varepsilon_0 r^2}$$

given $\sigma = \frac{q}{A} = C/m^2$

$$\sigma = \frac{q}{4\pi R^2}$$

$$q = 4\pi R^2 \sigma.$$

$$E = \frac{\cancel{4\pi} R^2 \sigma}{\cancel{4\pi} \varepsilon_0 r^2}$$

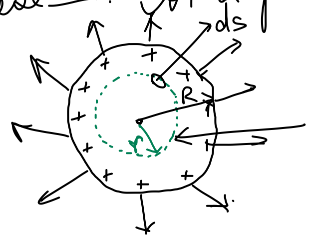
$$\boxed{E = \frac{\sigma R^2}{\varepsilon_0 r^2}}$$

$$\boxed{\vec{E} = \frac{\sigma R^2}{\varepsilon_0 r^2} \hat{r}}$$

Case II At a point on the surface of the shell $(r = R)$.

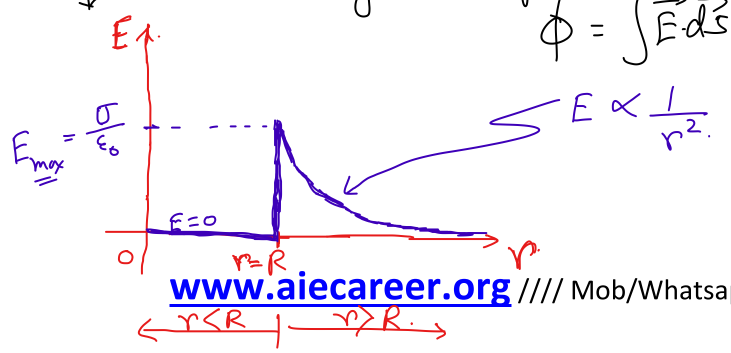
$$E = \frac{\sigma R^2}{\varepsilon_0 R^2} = \frac{\sigma}{\varepsilon_0} \quad \therefore \quad \boxed{E = \frac{\sigma}{\varepsilon_0} \qquad \vec{E} = \frac{\sigma}{\varepsilon_0} \hat{r}}$$ Constant

Case III At a point inside the shell $(r < R)$



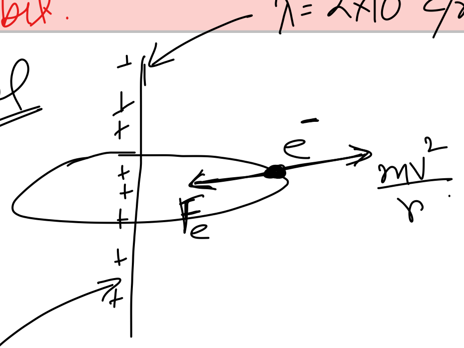
This gaussian plane does not enclose any charge, as the charges are always present at the surface. $\therefore q = 0$

$$\phi = \int \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0} = 0 \rightsquigarrow \boxed{E = 0}$$



An electron is revolving around a long line charge having charage density 2×10^{-8} C/m. Find the K.E of the electron. assume that it is indepedent of the radius of electron orbit.

Sol



$$E = \frac{1}{4\pi\varepsilon}\frac{2\lambda}{r}$$

When electron is revolving around the +ve charge density of line then the electrostatic force will balance the centripetal force

$$F_e = \frac{mv^2}{r}$$

$$qE = \frac{mv^2}{r}$$

$$q\left(\frac{2\lambda}{4\pi\varepsilon \cancel{r}}\right) = \frac{mv^2}{\cancel{r}}$$

$$mv^2 = \frac{2q\lambda}{4\pi\varepsilon}$$

$$K.E = \frac{1}{2}mv^2 = \frac{q\lambda}{4\pi\varepsilon}$$

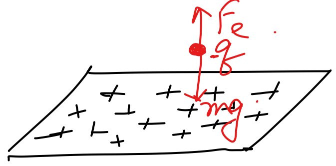
$$= 1.6\times10^{-19}\times2\times10^{-8}\times9\times10^{9}$$

$$= 28.8\times10^{-19-8+9}$$

$$= 28.8\times10^{-18}\ J.$$

Q A particle of mass 9×10^{-5} g is kept over a large horizontal sheet of charge density $+5\times10^{-5}$ c/m^2. What charge should be given to the particle, so that if released, it does not fall?

Sol $m = 9\times10^{-5}$ g



$$F_e = mg = qE$$

For sheet of charge $E = \frac{\sigma}{2\varepsilon_0}$

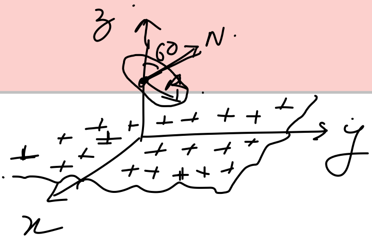
$$\Rightarrow mg = q\left(\frac{\sigma}{2\varepsilon_0}\right)$$

$$q = \frac{2mg\varepsilon_0}{\sigma}$$

$$q = \frac{2 \times 9\times10^{-5} \times 9.8 \times 8.85\times10^{-12} \times 10^{-3}}{5\times10^{-5}}$$

$$= 312.228 \times 10^{\cancel{-5}-12-3+\cancel{5}} = 312.228\times10^{-15}\ C$$

Q A large plane sheet of charge having surface charge density $5\times10^{-16}\ C/m^2$ lies in the x-y plane. Find the electric flux through a circular area of radius 0.1 m, if the normal to the circular area makes an angle of 60° with the z axis.



$R = 0.1\ m$

Sol $\sigma = 5\times10^{-16}\ C/m^2$

$\theta = 60°$

For sheet

$$E = \frac{\sigma}{2\varepsilon_0}$$

$$\phi = \vec{E}\cdot\vec{ds}$$

$$\phi = E\,ds\,\cos\theta$$

$$\phi = \frac{\sigma}{2\varepsilon_0}\cdot \pi R^2 \cdot \cos 60°$$

$$\phi = \frac{5\times10^{-16} \times 22 \times 0.1\times0.1 \times 1}{2 \times 7 \times 2\times 8.85\times10^{-12}}$$

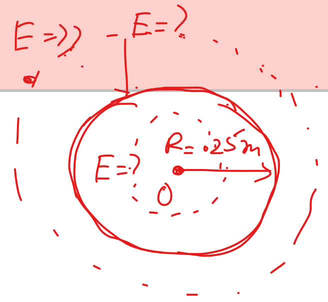
$$= \frac{5\times22\times0.01}{4\times7\times8.85} \times 10^{-16+12}$$

$$= 4.44\times10^{-3} \times 10^{-4}$$

$$\phi = 4.44\times10^{-7}\ Wb$$ Ans

Q. A spherical shell of metal has a radius of 0.25 m and carries a charge of 0.2 μC. Calculate the electric field intensity at a point.

1. Inside the shell.
2. Just outside.
3. 3 m from the Center of the shell.



Sol $q = 0.2\mu C$.

1. Inside $\rightarrow E = 0$.

2. $r = R. \longrightarrow E = \dfrac{1}{4\pi\varepsilon}\dfrac{q}{R^2}$

$$= 9\times10^9 \times \frac{0.2\times10^{-6}}{(0.25)^2} = \frac{9\times0.2}{0.25\times.25} \times 10^{9-6}$$

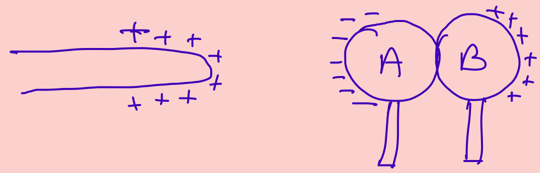
$$= \frac{1.8}{0.0625}\times10^3 = \frac{1800}{0.0625} = \frac{1800}{625}\times10^4$$

$$= 2.88\times10^4\ N/C.\ \text{Ans}$$

3. $r = 3m$ $\quad E = \dfrac{1}{4\pi\varepsilon}\dfrac{q}{r^2} = 9\times10^9 \times \dfrac{0.2\times10^{-6}}{(3)^2} = 200 N/c.$

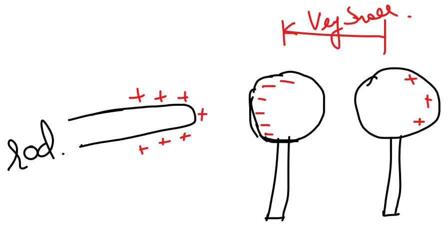
A glass rod rubbed with silk is brought close to two uncharged sphere in contact with each other as shown.

Describe what happens when.



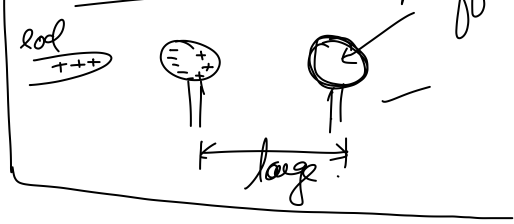
1. spheres are slightly separated.
2. The glass rod is subsequently removed and finally.
3. The spheres are separated far apart?

Sol ①

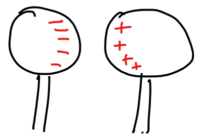


When the spheres are separated by a very short distance, there is very little change in the distribution of charge.

NOTE

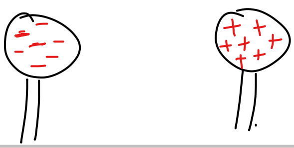


② No rod



When rod removed, there will be distribution of charges i.e the +ve and −ve will be attracted as shown.

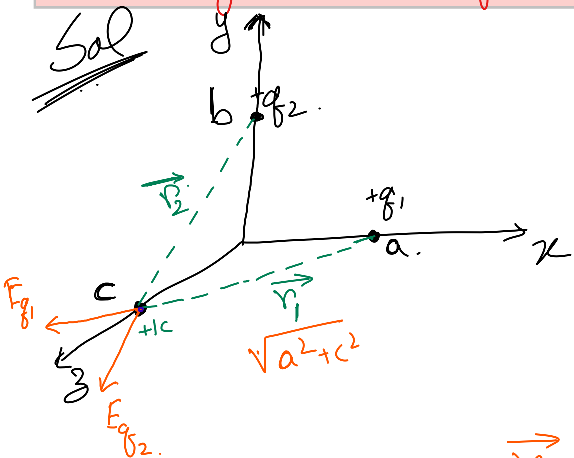
③



• Charges gets Uniformly distributed.

Q Two point charges q_1 and q_2 are located at points $(a,0,0)$ and $(0,b,0)$ resp. Find the electric field due to both the charges at the point $(0,0,c)$.

Sol



$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}\hat{a}_n}$$

$\vec{r} \rightarrow$ distance vector.

To find r_1 → distance Vector.

$(0,0,c)$ $(a,0,0)$

$\vec{r_1} = (0-a)\hat{i} + (0-0)\hat{j} + (c-0)\hat{k}$

$\boxed{\vec{r_1} = -a\hat{i} + c\hat{k}}$

$|r_1| = \sqrt{a^2+c^2}$

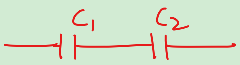
To find r_2

$(0,0,c)$ $(0,b,0)$

$\vec{r_2} = (0-0)\hat{i} + (0-b)\hat{j} + (c-0)\hat{k}$

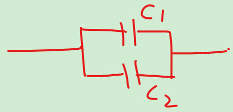
$\boxed{\vec{r_2} = -b\hat{j} + c\hat{k}}$ $|r_2| = \sqrt{b^2+c^2}$

Capacitor.



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

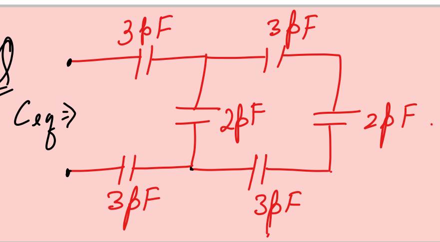
- Voltage gets divided.
- Charge remain same.



$$C_{eq} = C_1 + C_2 .$$

- Voltage remains same.
- Charge gets divided.

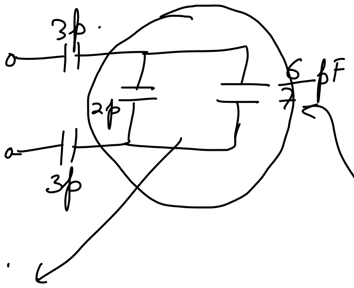
Q



The eq Capacitance at the Input is

a) $\frac{24}{60}$ pF b) $\frac{60}{61}$ pF c) $\frac{64}{61}$ pF d) $\frac{24}{67}$ pF

Sol



$$\frac{1}{C} = \frac{1}{3} + \frac{1}{2} + \frac{1}{3}$$

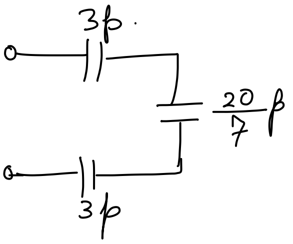
$$\frac{1}{C} = \frac{2+3+2}{6} = \frac{7}{6} .$$

$$C = \frac{6}{7} \text{ pF}$$

parallel.

$$C = 2p + \frac{6}{7} p .$$

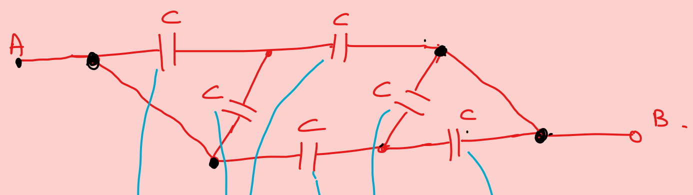
$$= \frac{14+6}{7} = \frac{20}{7} \text{ pF} .$$



$$\frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{20/7} = \frac{2}{3} + \frac{7}{20} = \frac{60}{61}$$

$$C_{eq} = \frac{60}{61} \text{ pF} \quad \text{Ans}$$

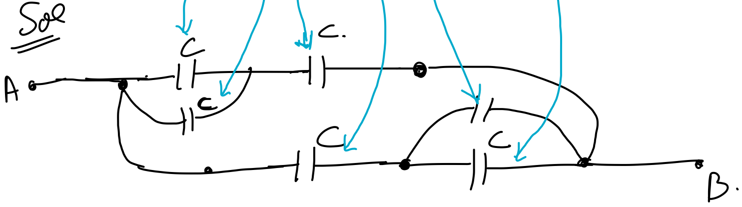
Q

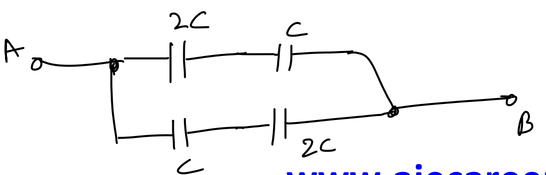


The Eq Capacitance will be.

a) $\frac{4C}{3}$ b) $\frac{2C}{3}$ c) $\frac{C}{2}$ d) $\frac{3C}{4}$.

Sol

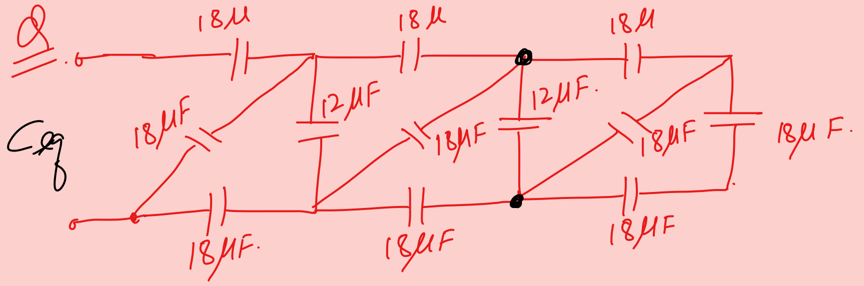




$$C_{eq} = \left(\frac{4C}{3}\right)$$

Q.

C_{eq}



The Eq Cap.

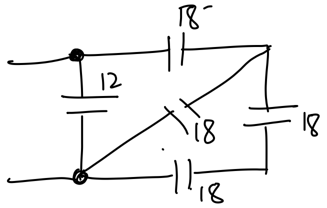
a). 22μF

b) 33μF

c) 11μF

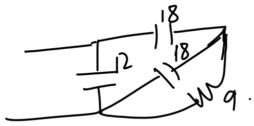
d) 44μF

Sol

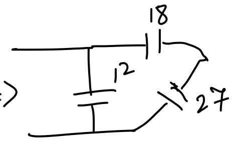


$\frac{1}{C} = \frac{1}{18} + \frac{1}{18}$

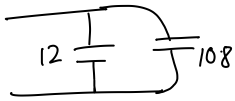
$C = 9$



$\Rightarrow$



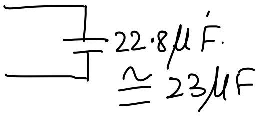
$\Rightarrow$



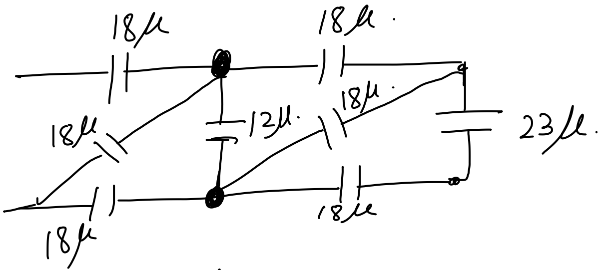
$\frac{1}{C} = \frac{1}{18} + \frac{1}{27}$

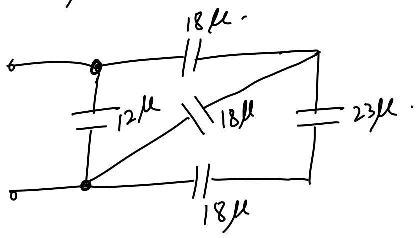
$C = \frac{18 \times 27}{18 + 27}$

$= \frac{54}{5}$ or $10.8\mu F$



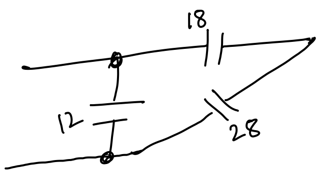
$12 + 10.8$





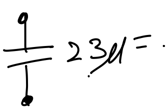
$\Rightarrow \frac{18 \times 23}{18 + 23} + 18 = \frac{414}{41} + 18$

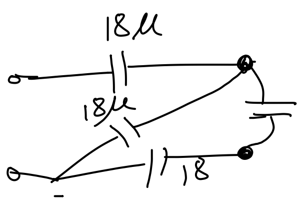
$= \frac{1152}{41} = 28.09 \simeq 28$



$\left(\frac{18 \times 28}{18 + 28}\right) + 12 = \left(\frac{252}{23}\right) + 12 = \frac{526}{23} = 22.96$

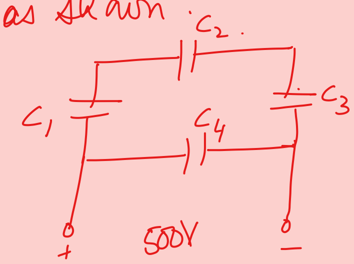
$\simeq 23$





$= \frac{\cancel{253}}{\cancel{23}} = 11\,\Omega$

Q) A network of four capacitors each of 12 μF capacitance is connected to a 500V supply as shown.



(1) The Eq Cap of the network

a) 20 μF
b) 24 μF
c) 16 μF
d) 12 μF.

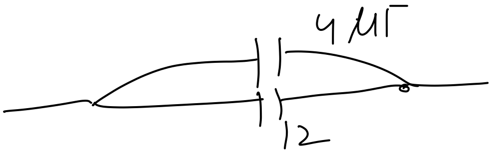
(2) Charge on each Capacitor.

	q_1	q_2	q_3	q_4
a)	6000 μC	6000 μC	6000 μC	2000 μC
b)	400 μC	200 μC	200 μC	300 μC
c)	2000 μC	2000 μC	2000 μC	6000 μC
d)	1000 μC	1000 μC	1000 μC	2000 μC

Sol (1)

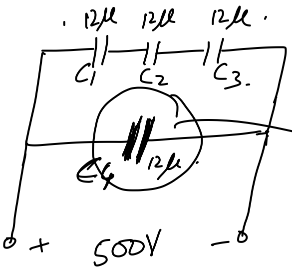
$$\frac{1}{C_{series}} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{12}$$

$C_{ser} = 4\ \mu F$



$C_{eq} = 12\mu + 4\mu = 16\ \mu F.$

(2)



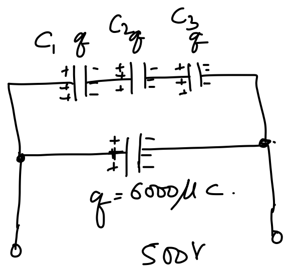
$\boxed{q = CV}$

For C_4.

$q = C \cdot V$

$q = 12\mu \times 500$

$q_{C4} = \underline{6000\ \mu C}.$



$C' \rightarrow$ Series of $C_1, C_2 \& C_3$

$$\frac{1}{C'} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$$

$C' = 4\ \mu F.$

$q = CV.$

$= C' \times 500$

$= 4 \times 500\ \mu.$

$q = 2000\ \mu C.$

Since the Cap s are connected in series. ∴ will have same charge

$q = 2000\ \mu C = q_1 = q_2 = q_3$ Ans

Energy in Cap :- $\boxed{U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV}$ $\longrightarrow$ Unit $\rightarrow$ J.

Energy stored per unit Volume $\Rightarrow$ $U = \frac{1}{2}\varepsilon_0 E^2$ $\longrightarrow$ Unit J/m³.

Q A parallel plate capacitor of 300μF is charged to 200V. If the distance between its plates is halved, what will be the P.D between the plates? what will be the change in stored energy?

Sol

c , d ; c' , $d/2$

$q = cV = 300\mu \times 200$

$C = \frac{\varepsilon_0 A}{d}$; $C' = \frac{\varepsilon_0 A}{d'}$

$\frac{C}{C'} = \frac{\varepsilon_0 A/d}{\varepsilon_0 A/d'} = \frac{d'}{d}$

$\boxed{\frac{C}{C'} = \frac{d'}{d}}$ $\Rightarrow$ $\frac{C}{C'} = \frac{d/2}{d} = \frac{1}{2}$

$C' = 2C = 2 \times 300 = 600\,\mu F$

$q = \frac{300 \times 200}{1000000} = 0.06\,C$

charge remains same.

$q = C'V'$

$V' = \frac{0.06}{600} \times 1000000$

$V' = \frac{600}{6} = 100V$ Ans

$U_1 = \frac{1}{2}CV^2 = 6J$

$U_2 = \frac{1}{2}C'(V')^2 = 3J.$

$\therefore \Delta U = U_2 - U_1 = 3 - 6 = -3J$ Ans

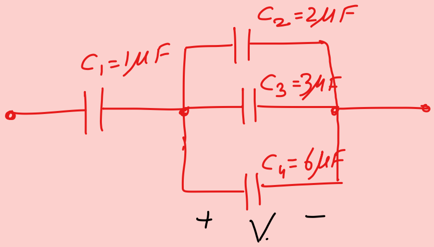
Q For the fig shown :- The energy stored in C_4 is 27 J. The Total energy stored in the system is

a) 594 J.

b) 560 J.

c) 420 J.

d) 730 J



Sol $E_{C_4} = \frac{1}{2}C_4V^2$

$27 = \frac{1}{2} \times 6\mu \times V^2$

$V^2 = \frac{\cancel{27}^{9} \times \cancel{2} \times 1000000}{\cancel{6}\,3}$

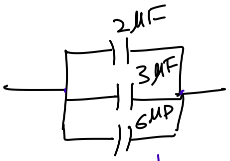
$V = 3 \times 1000 = 3000V.$

The voltage of C_2, C_3 and C_4 are same as they are connected in parallel. $V = 3000V$.

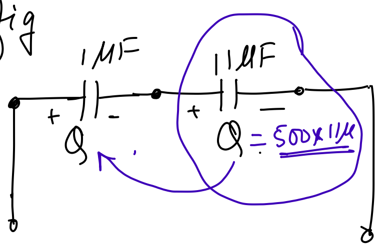
$E_{C_3} = \frac{1}{2}C_3V^2 = \frac{27}{2}$ J

$E_{C_2} = \frac{1}{2}C_2V^2 = 9J.$

$\therefore$ Energy in $C_2, C_3 \& C_4 \Rightarrow 27 + 9 + 13.5$
$= 49.5J.$



original fig



$Q = CV$.

$= 2\mu \times 500 + 3\mu \times 500 + 6\mu \times 500$.

$Q_{Total} = 11\mu \times 500\ C$.

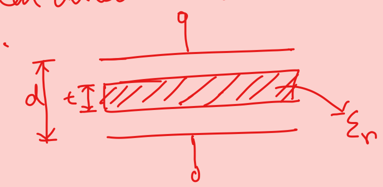
$\therefore$ charge is C_1 is equal to 11µF as they are connected in series. (charge remains same)

$$U_{C_1} = \frac{1}{2}\frac{Q^2}{C}.$$

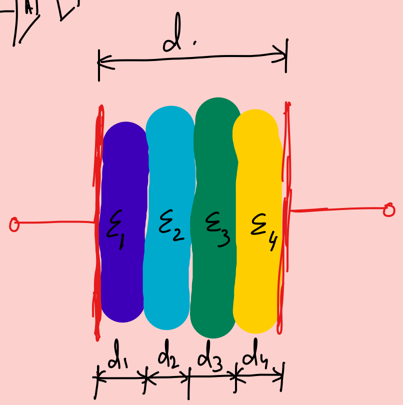
$$= \frac{1}{2}\,\frac{11\mu \times 500 \times 11\mu \times 500}{1\mu}$$

$$= \frac{121 \times \cancel{500} \times \cancel{500}}{2 \times \cancel{1000000}_{4}} = \frac{121}{8} = 15.125\ J.$$

- Capacitor with Air dielectric. $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$; $\varepsilon_r = 1$ $\therefore$ $\boxed{C = \frac{\varepsilon_0 A}{d}}$.
- when a dielectric is present $\boxed{C = \frac{\varepsilon_0 \varepsilon_r A}{d}.}$ $\varepsilon_r = K$
- when dielectric is a slab of ε_r.

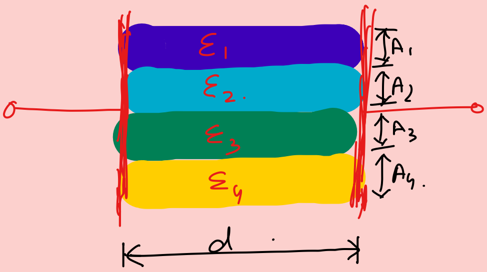


$$\boxed{C = \frac{\varepsilon_0 A}{d - t + \frac{t}{\varepsilon_r}}}$$



- Series type arragement.

$$\boxed{C = \frac{\varepsilon_0 A}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} + \frac{d_3}{\varepsilon_3} + \frac{d_4}{\varepsilon_4}}}$$



- parallel type arragement.

$$\boxed{C = \frac{\varepsilon_0}{d}\left(\varepsilon_1 A_1 + \varepsilon_2 A_2 + \varepsilon_3 A_3 + \varepsilon_4 A_4\right)}$$