Matrices

It is an rectangular array of m rows $\leftrightarrow$ and n Columbs $\updownarrow$.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ & \vdots & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

$m \times n \leftarrow$ order of Matrix.

$a_{mn} \rightarrow a_{ij}$ — rows, Col.

$a_{32} \rightarrow$ 3rd Row and 2nd Col.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -1 & -2 \end{bmatrix}$$

Rows − 2.
Col − 3.
order = 2×3.

$a_{22} \rightarrow -1$.
$a_{12} \rightarrow 2$.
$a_{33} \rightarrow 0$

Q Construct a 3×4 matrix whoes element Values are ① $a_{ij} = i + j$.

② $a_{ij} = \frac{2i - j3}{4}$.

Sol ① $= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$ Ans

② $\begin{bmatrix} -1/4 & -1 & -7/4 & -5/2 \\ 1/4 & -1/2 & -5/4 & -2 \\ 3/4 & 0 & -3/4 & -3/2 \end{bmatrix}$

Types of Matrices.

① Column Matrix $= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}_{3 \times 1}$

② Row Matrix $= \begin{bmatrix} a & b & c \end{bmatrix}_{1 \times 3}$.

③ Square Matrix $= \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2 \times 2}$ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3 \times 3}$.

④ Diagonal Matrix $= \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

⑤ Scalar Matrix $= \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \longrightarrow$ $a_{ij} = 0 \ : i \neq j$
$a_{ij} = a \ : i = j$

⑥ Unit or Identity Matrix $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow$ $a_{ij} = 0 \ : i \neq j$
$a_{ij} = 1 \ : i = j$

⑦ Null Matrix. $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

⑧ Upper Triangular Matrix $= \begin{bmatrix} 1 & 4 & 6 & 5 \\ 0 & 2 & 1 & 3 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 0 & 7 \end{bmatrix} \longrightarrow$ $a_{ij} = 0 \ : i > j$

(9) Lower Triangular Matrix $= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 5 & 0 & 0 \\ 3 & 6 & 8 & 0 \\ 4 & 7 & 9 & 1 \end{bmatrix} \longrightarrow a_{ij} = 0 \;:\; i<j$

Q Two matrices. $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{r\times s}$.
give Comment for the Conditions.
(1) $m = r$.
(2) $n = s$.
(3) $a_{ij} = b_{ij}$.

Sol If the above Cond are all together then matrix A and B are called equal Matrix.

2013
Q If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ Find x, y, z and w.

Sol
$x - y = -1$
$2x + z = 5$
$2x - y = 0$
$3z + w = 13$.

Solve to get $x = 1$
$y = 2$
$z = 3$
$w = 4$

Q A matrix has 12 elements. What are the possible orders it can have?

Sol 1×12, 12×1, 3×4, 4×3, 2×6, 6×2.

Total — 6 orders.

Addition and Substraction of Matrix.

If $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$.

$$(A \pm B)_{ij} = a_{ij} \pm b_{ij}$$

NOTE possible only when orders are same.

Properties of Matrix Addition:–

(1) $A + B = B + A$ — Commutative law.

(2) $(A + B) + C = A + (B + C)$ — Associative law.

(3) $A + O = O + A$. — Identity law.

(4) $A + (-A) = O = (-A) + A$. — Inverse law.

(5) $A + B = A + C \Rightarrow B = C$. — Cancellation law.

SCALAR MULTIPLICATION

$$KA = [K a_{ij}]_{m\times n}$$

Ex $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 2 \\ 4 & 5 & 0 \end{bmatrix}$ Now $\frac{10}{3}A = \begin{bmatrix} \frac{10}{3} & \frac{20}{3} & 10 \\ -\frac{10}{3} & 10 & \frac{20}{3} \\ \frac{40}{3} & \frac{50}{3} & 0 \end{bmatrix}$.

Property of Scalar multiplication

If $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{m\times n}$ and K and l are Scalar.

① $K(A \pm B) = KA \pm KB$.

② $(K \pm l)A = KA \pm lA$.

③ $(Kl)A = K(lA) = l(KA)$

④ $(-K)A = -(KA) = K(-A)$

⑤ $1A = A$.

⑥ $(-1)A = -A$.

Multiplication of Matrices.

— Row Column Multiplication.

$$[\quad]_{m\times n} \times [\quad]_{a\times b} = [\quad]_{m\times b}$$

ax ay.

Ex → $\begin{bmatrix} a \\ b \\ c \end{bmatrix}_{3\times 1} [x \ y \ z]_{1\times 3} = \begin{bmatrix} ax & ay & az \\ bx & by & bz \\ cx & cy & cz \end{bmatrix}_{3\times 3}$.

$\begin{bmatrix} a & 0 & 0 \\ b & 0 & 0 \\ c & 0 & 0 \end{bmatrix}_{3\times 3} \begin{bmatrix} x & y & z \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3} = \begin{bmatrix} ax & ay & az \\ bx & by & bz \\ cx & by & cz \end{bmatrix}$

$[x \ y \ z]_{1\times 3} \begin{bmatrix} a \\ b \\ c \end{bmatrix}_{3\times 1} = [ax + by + cz]_{1\times 1}$

→ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} a\times1 + b\times4 + c\times7 & a\times2 + b\times5 + c\times8 & a\times3 + b\times6 + c\times9 \\ d\times1 + e\times4 + f\times7 & d\times2 + e\times5 + f\times8 & d\times3 + e\times6 + f\times9 \\ g\times1 + h\times4 + i\times7 & g\times2 + h\times5 + i\times8 & g\times3 + h\times6 + i\times9 \end{bmatrix}$

$$= \begin{bmatrix} a+4b+7c & 2a+5b+8c & 3a+6b+9c \\ d+4e+7f & 2d+5e+8f & 3d+6e+9f \\ g+4h+7i & 2g+5h+8i & 3g+6h+9i \end{bmatrix}$$

Properties

1. $AB \neq BA \longrightarrow$ not commutative.
2. $(AB)C = A(BC)$. $\rightarrow$ associative.
3. $A(B+C) = AB + AC$. or. $(A+B)C = AC + BC$. — distributive.
4. $IA = AI$

Power
1. $A^{n+1} = A^n \cdot A$. $\quad n \in N$

$A^2 = A \cdot A$.
$A^3 = A^2 A$.

2. $A^m A^n = A^{m+n}$.
3. $(A^m)^n = A^{mn}$.

Q If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$

Prove $(A+B)^2 \neq A^2 + B^2 + 2AB$.

Sol $A+B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$(A+B)^2 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 18 & 9 \end{bmatrix}$. $\longrightarrow$ LHS.

$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$.

$B^2 = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$

$2AB = 2\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 14 & 4 \end{bmatrix}$

$A^2 + B^2 + 2AB$.

$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ 14 & 4 \end{bmatrix}$

$= \begin{bmatrix} 6 & 0 \\ 24 & 12 \end{bmatrix} \longrightarrow$ RHS $\quad \therefore$ RHS $\neq$ LHS.

Q If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ find x and y such that $(xI + yA)^2 = A$.

Sol $(xI + yA)$

$x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + y\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$

$(xI + yA)^2 = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}\begin{bmatrix} x & y \\ -y & x \end{bmatrix}$

$\therefore \begin{bmatrix} x^2 - y^2 & 2xy \\ -2xy & x^2 - y^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = A$.

$x^2 - y^2 = 0$. $\quad 2xy = 1$

$x = \pm y$.

Case I $x = y$.
$2x^2 = 1$
$x^2 = \frac{1}{2}$.
$x = \pm \frac{1}{\sqrt{2}}$

Case II $x = -y$.
$-2x^2 = 1$
$x^2 = -\frac{1}{2}$.
$x \rightarrow$ Imaginary.

$x = \frac{1}{\sqrt{2}}$; $y = \frac{1}{\sqrt{2}}$ and $x = -\frac{1}{\sqrt{2}}$; $y = -\frac{1}{\sqrt{2}}$ Ans

Q Let $A = \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I be the identity matrix of order 2. Show that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

Sol

$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 1 \end{bmatrix} \rightarrow \text{RHS}.$$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}$$

$$\cos\alpha = \frac{1 - \tan^2\alpha/2}{1 + \tan^2\alpha/2}$$

$$\sin\alpha = \frac{2\tan\alpha/2}{1 + \tan^2\alpha/2}$$

$$\tan\alpha/2 = x$$

$$= \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & x \\ -x & 1 \end{bmatrix}\begin{bmatrix} \frac{1-x^2}{1+x^2} & -\frac{2x}{1+x^2} \\ \frac{2x}{1+x^2} & \frac{1-x^2}{1+x^2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1-x^2+2x^2}{1+x^2} & \frac{-2x+x-x^3}{1+x^2} \\ \frac{-x+x^3+2x}{1+x^2} & \frac{2x^2+1-x^2}{1+x^2} \end{bmatrix} = \begin{bmatrix} \frac{1+x^2}{1+x^2} & \frac{-x(x^2+1)}{1+x^2} \\ \frac{x(1+x^2)}{(1+x^2)} & \frac{1+x^2}{1+x^2} \end{bmatrix} = \begin{bmatrix} 1 & -x \\ x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 1 \end{bmatrix} \longrightarrow \text{LHS} = \text{RHS}.$$

Q Find the value of x such that $[1, x, 1]\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$.

Sol

$$\underbrace{\begin{bmatrix} 1 & x & 1 \end{bmatrix}}_{1\times3}\underbrace{\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}}_{3\times3}\underbrace{\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}}_{3\times1} = 0$$

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1+6+2x \\ 2+10+x \\ 15+6+2x \end{bmatrix} = 0 \Rightarrow \underbrace{\begin{bmatrix} 1 & x & 1 \end{bmatrix}}_{1\times3}\underbrace{\begin{bmatrix} 7+2x \\ 12+x \\ 21+2x \end{bmatrix}}_{3\times1} = 0$$

$$(7+2x)1 + (12+x)x + 1(21+2x) = 0$$

$$7 + 2x + 12x + x^2 + 21 + 2x = 0$$

$$x^2 + 16x + 28 = 0.$$

$$x^2 + 2x + 14x + 28 = 0$$

$$x(x+2) + 14(x+2) = 0 \rightarrow x+2 = 0,\ x = -2 \quad \Big| \quad x + 14 = 0,\ x = -14$$

Ans

Q If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$. Find A.

Sol

$3\times 2 \quad [A]_{2\times 3} = 3\times 3.$

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}_{2\times 2} \begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix}_{2\times 3} = \begin{bmatrix} & & \end{bmatrix}_{3\times 3}$$

$$\begin{bmatrix} 2x-a & 2y-b & 2z-c \\ x & y & z \\ -3x+4a & -3y+4b & -3z+4c \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$$

Q Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$. Use this result to find A^5.

Sol

$A^2 = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$

$-4A = \begin{bmatrix} -8 & -12 \\ 4 & -8 \end{bmatrix}$

$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} + \begin{bmatrix} -8 & -12 \\ 4 & -8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ proved.

$A^2 - 4A + 7 = 0$

$A^2 = 4A - 7$.

$A^3 = A^2 \cdot A = (4A-7)A$.

$= 4A^2 - 7A$.

$A^3 = 4[4A-7] - 7A$

$= 16A - 28 - 7A$

$A^3 = 9A - 28$.

$A^4 = A^3 \cdot A$.

$= [9A-28]A$.

$= 9A^2 - 28A$.

$= 9[4A-7] - 28A$.

$= 36A - 63 - 28A$.

$A^4 = 8A - 63$

$A^5 = A^4 \cdot A$

$= (8A-63)A$.

$= 8A^2 - 63A$.

$= 8[4A-7] - 63A$

$= 32A - 56 - 63A$.

$A^5 = -31A - 56I$.

$A^5 = -31\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 56\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$

62
56
118

TRANSPOSE of MATRIX :-

Transpose A^T or A' $\quad (A^T)_{ij} = a_{ji}$.

Properties - ① $(A^T)^T = A$.

② $(A+B)^T = A^T + B^T$.

③ $(KA)^T = K(A)^T$.

④ $(AB)^T = A^T . B^T$.

Q If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ then find the value of θ satisfying the equation $A^T + A = I$.

Sol $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2\cos\theta & 0 \\ 0 & 2\cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$2\cos\theta = 1 \quad \therefore \cos\theta = 1/2$.

$\cos\theta = \cos \pi/3$. $\quad \theta = \pi/3$ Ans

SYMMETRIC AND SKEWSYMMETRIC MATRIX.

If $a_{ij} = a_{ji}$ for all i, j then symmetric. Eg $a_{13} = a_{31}$

$a_{12} = a_{21}$

$\begin{bmatrix} a & b & c \\ x & y & z \\ \alpha & \beta & \gamma \end{bmatrix}$ Symmetric $\begin{bmatrix} a & x & \alpha \\ b & y & \beta \\ c & z & \gamma \end{bmatrix}$ $\quad \boxed{A_{ij} = (A^T)_{ij}}$

SKEW SYMMETRIC MATRIX.

$A = a_{ij}$ then for skew sym matrix $a_{ij} = -a_{ji}$

$A = \begin{bmatrix} 0 & 2i & 3 \\ -2i & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 2i & +3 \\ -2i & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix}$ same.

$A^T = \begin{bmatrix} 0 & -2i & -3 \\ 2i & 0 & -4 \\ 3 & 4 & 0 \end{bmatrix} \Rightarrow - \begin{bmatrix} 0 & 2i & 3 \\ -2i & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix}$

$\boxed{A^T = -A \text{ if skew Symmetric.}}$

Q If A is a square matrix. Then
① $A+A^T$ is Symmetric matrix.
② $A-A^T$ is skew symmetric Matrix.

Sol Let $P = A + A^T$.

$P^T = (A+A^T)^T$.

$P^T = A^T + (A^T)^T$

$P^T = A^T + A = P$

$P^T = P$.

P is Symmetric matrix.

② $Q = A - A^T$.

$Q^T = (A-A^T)^T$.

$Q^T = A^T - (A^T)^T$

$Q^T = A^T - A$.

$Q^T = -(A-A^T) = -Q$.

$Q^T = Q$.

Q is skew symmetric.

Q Prove that every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew Sym matrix.

Sol Let A be a square matrix.

$A = P + Q$. where $P = \frac{1}{2}(A+A^T)$ and $Q = \frac{1}{2}(A-A^T)$

$P^T = \left(\frac{1}{2}(A+A^T)\right)^T$.

$= \frac{1}{2}(A+A^T)^T$.

$= \frac{1}{2}(A^T+A)$.

$P^T = P$. Sym. Similary $Q^T = -Q$. skew Sym.

Hence, $A = P + Q$ which is the sum of Sym and skew Sym matrix.

Q Show $(B)^T AB$ is sym or skew Sym according to A is Sym or skew Sym.

Sol Case I A be symmetric. $A^T = A$.

$(B^TAB)^T = B^T A^T B$

$(B^TAB)^T = (B^T AB)$.

B^TAB is symmetric.

Case II A is skew Symmetric. $A^T = -A$.

$(B^TAB)^T = B^T A^T B$.

$= -B^T A B$.

$(B^TAB)^T = -(B^TAB)$

B^TAB is skew Sym.

Q Show that all possible Integral powers of a symmetric matrix are Symmetric.

Sol:

$A^n = A\cdot A\cdot A \dots n$ times.

$(A^n)^T = (A\cdot A\cdot A \dots n\text{times})^T$.

$(A^n)^T = A^T A^T A^T \dots n$ time.

$(A^n)^T = (A^T)^n$.

For Sym $A^T = A$.

$(A^n)^T = A^n$ Ans

NOTE

For Skew Sym. $A^T = -A$.

$(A^n)^T = (-A)^n$.

$(A^n)^T = (-1)^n A^n$.

$(A^n)^T \to A^n$: n is even.

$(A^n)^T \to -A^n$: n is odd.

Q Express the matrix $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ as the sum of symmetric and a skew symmetric matrix.

Sol

$P = \frac{1}{2}(A + A^T)$

$Q = \frac{1}{2}(A - A^T)$

$A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ ∴ $A^T = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix}$.

$A + A^T = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 10 & 7 \\ 5 & 7 & 10 \end{bmatrix}$

$A - A^T = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

$P = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix} = \frac{(A + A^T)}{2}$

Sym

$P^T = P \rightarrow \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix}$

$Q = \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix} = \frac{A - A^T}{2}$.

Skew Sym.

$Q^T = -Q \Rightarrow Q^T = \begin{bmatrix} 0 & 1 & -1/2 \\ -1 & 0 & 1/2 \\ 1/2 & -1/2 & 0 \end{bmatrix}$

$-Q^T = \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix} = Q$.

$P + Q = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$

$\boxed{P + Q = A}$

Elementary operation of Matrices.

① Interchange of Row or Column.

$$A = \begin{bmatrix} 1 & 2 & \sqrt{2} \\ 3 & -5 & 4 \\ x & y & 10 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$. $= \begin{bmatrix} 1 & 2 & \sqrt{2} \\ x & y & 10 \\ 3 & -5 & 4 \end{bmatrix}$

$C_1 \leftrightarrow C_3$ $= \begin{bmatrix} \sqrt{2} & 2 & 1 \\ 4 & -5 & 3 \\ 10 & y & x \end{bmatrix}$

② Multiplication.

$R_2 \to 3R_2 . \Rightarrow \begin{bmatrix} 1 & 2 & \sqrt{2} \\ 9 & -15 & 12 \\ x & y & 10 \end{bmatrix}$

$C_3 \to -4C_3 . \Rightarrow \begin{bmatrix} 1 & 2 & -4\sqrt{2} \\ 3 & -5 & -16 \\ x & y & -40 \end{bmatrix}$

③ Add/Sub of Row and Col.

$R_1 \to R_1 + R_3 \Rightarrow \begin{bmatrix} 1+x & 2+y & \sqrt{2}+10 \\ 3 & -5 & 4 \\ x & y & 10 \end{bmatrix}$

$C_2 \to C_2 - C_1$ $\begin{bmatrix} 1 & 1 & \sqrt{2} \\ 3 & -8 & 4 \\ x & y-x & 10 \end{bmatrix}$.

④ Mul of Scalar with Row/Col.

$R_2 \to R_2 - \frac{R_1}{2}$ $\begin{bmatrix} 2 & 4 & 3 \\ 1 & 5 & 2 \\ 3 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 3 & 1/2 \\ 0 & -2 & 5 \end{bmatrix}$

$R_3 \to R_3 - \frac{3R_1}{2}$

$\to R_2 \to$ $1 - \frac{1}{\cancel{2}}\cancel{2} = 0$

$5 - \frac{1}{\cancel{2}} \cdot \cancel{4}^{2} = 3$

$2 - \frac{1}{2} \cdot 3 = 1/2$.

$3 - \frac{3}{\cancel{2}} \cdot \cancel{2} = 0$

$4 - \frac{3}{\cancel{2}} \cdot \cancel{4}^{2} = -2$.

$7 - \frac{3}{2} \cdot 3 = 5$

$R_3 \to R_3 + \frac{2R_2}{3}$.

$0 + \frac{2}{3} \cdot 0 = 0$

$-2 + \frac{2}{3}(\cancel{3}) = 0$

$5 + \frac{\cancel{2}}{3}(\frac{1}{\cancel{2}}) = \frac{16}{3}$

$\begin{bmatrix} 2 & 4 & 3 \\ 0 & 3 & 1/2 \\ 0 & 0 & 16/3 \end{bmatrix}$

No of non zero Rows $- 3$.

$-$ Rank.

If square matrix A of order n is said to be invertible if there exists a square matrix B of order n such that.

$\boxed{AB = BA = I}$ $\boxed{A^{-1} = B}$

Ex $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$.

$A \cdot A^{-1} = I$.
$A \cdot B = I$
$B = A^{-1}$

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ $AB = BA = I$

$B \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

B matrix is Invertible of A

$\boxed{A^{-1} = B.}$

Q

$A \times B = I$.

$\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 3a+5c. & 3b+5d. \\ a+2c. & b+2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$

$3b + 5d = 0$
$d = -\frac{3b}{5}$
$d = 3$
$3a + 5c = 1$
$3a + 5\left(-\frac{a}{2}\right) = 1$
$a\left(3 - \frac{5}{2}\right) = 1$
$a\left(\frac{1}{2}\right) = 1$
$a = 2$

$a + 2c = 0$.
$c = -\frac{a}{2}$.
$c = -1$
$b + 2d = 1$
$b + 2\left(-\frac{3b}{5}\right) = 1$
$b\left[1 - \frac{6}{5}\right] = 1$
$b\left(-\frac{1}{5}\right) = 1$
$b = -5$

Ex 3.1. Q 10

Q. The No. of all possible matrixes of order 3×3 with entry 0 or 1 is ______

Sol $A = \begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{bmatrix}_{3\times3}$ No. of element = 9.

0 1.

9 places can be filled in $2^9 = 512$ ways.

3.2 Q 21

Sol $X = [\;]_{2\times n}$.
$Y = [\;]_{3\times K}$.
$Z = [\;]_{2\times p}$.
$W = [\;]_{n\times3}$
$P = [\;]_{p\times K}$.

(21) $PY = WY$.

$[\;]_{p\times K}[\;]_{3\times K} = [\;]_{n\times3}[\;]_{3\times K}$.

$[\;]_{p\times K} = [\;]_{n\times K}$.

$p = n$: K $\longrightarrow$ (A) $\longrightarrow$ $p = n$ $K = 3$.

(22) $7[\ \ ]_{2\times n} - 5[\ \ ]_{2\times p} \Rightarrow$ has to be Same order $\therefore 2\times n \longrightarrow (B)$.

Q By Using elementary row operation, find the Inverse of Matrix = $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

Sol $[A] = [I][A]$

we will Convert this $[A]$ to $[I]$ $\longrightarrow [I] = [A^{-1}][A]$

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}[A]$$

$R_2 \to R_2 + 3R_1$

$\to -3 + 3(1) = 0$
$0 + 3(3) = 9$
$-5 + 3(-2) = -11$

$\to 0 + 3(1) = 3$
$1 + 3(0) = 1$
$0 + 3(0) = 0$

$R_3 \to R_3 - 2R_1$

$\to 2 - 2(1) = 0$
$5 - 2(3) = -1$
$0 - 2(-2) = 4$.

$\to 0 - 2(1) = -2$
$0 - 2(0) = 0$
$1 - 2(0) = 1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}[A]$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -1 & 4 \\ 0 & 9 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix} \cdot [A]$$

$R_1 \to R_1 + 3R_2$.

$\to 1 + 3(0) = 1$
$3 + 3(-1) = 0$
$-2 + 3(4) = 10$

$1 + 3(-2) = -5$
$0 + 3(0) = 0$
$0 + 3(1) = 3$.

$R_3 \to R_3 + 9R_2$.

$\to 0 + 9(0) = 0$
$9 + 9(-1) = 0$
$-11 + 9(4) = 25$

$\to 3 + 9(-2) = -15$
$1 + 9(0) = 1$
$0 + 9(1) = 9$.

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & -1 & 4 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -2 & 0 & 1 \\ -15 & 1 & 9 \end{bmatrix}[A]$$

$R_3 \to \frac{1}{25}R_3$

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -2 & 0 & 1 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}[A]$$

$R_1 \to R_1 - 10R_3$.

$1 - 10(0) = 1$
$0 - 10(0) = 0$
$10 - 10(1) = 0$
$-5 - \cancel{10}^{2}(-3/\cancel{5}) = -5 + 6 = 1$
$0 - 10(1/25) = -2/5$
$3 - \cancel{10}^{2}(9/\cancel{25}_5) = -3/5$.

$\bullet\ R_2 \to R_2 - 4R_3$.

$0 - 4(0) = 0$
$-1 - 4(0) = -1$
$4 - 4(1) = 0$
$-2 - 4(-3/5) = \frac{2}{5}$
$0 - 4(1/25) = -4/25$
$1 - 4(9/25) = \frac{-11}{25}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2/5 & -3/5 \\ 2/5 & -4/25 & -11/25 \\ -3/5 & 1/25 & 9/25 \end{bmatrix}[A]$$

$R_2 \to -R_2$.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2/5 & -3/5 \\ -2/5 & 4/25 & 11/25 \\ -3/5 & 1/25 & 9/25 \end{bmatrix}[A]$$

$I = A^{-1}A$

$A^{-1}[\ \ ]$

Q If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{bmatrix}$ show A^{-1} does not exist.

Sol

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 + R_1$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & -3 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A.$$

$R_3 \to R_3 + R_2.$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & -3 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} A$$

$\to$ This can never be made I Matrix.

$\therefore$ not possible to get A^{-1}.